Big Bang- Big Crunch Optimization in Second Order Sliding Mode Control

TejasKulkarni

Department of Electrical Engineering, G.H.Raisoni Institute of Engineering and Technology, Pune, India.

ABSTRACT

In this article, Second order sliding mode with Big Bang- Big Crunch optimization technique is employed for nonlinear uncertain system.The sliding surface describes the transient behavior of a system in sliding mode. Frequently, PD- type sliding surface is chosen as a hyperplane in the system state space.An integral term incorporated in the sliding surface expression that resulted in a type of PID sliding surface as hyperbolic function for alleviating chattering effect. The sliding mode control law is derived using direct Lyapunov stability approach and asymptotic stability is proved theoretically. Here, novel tuning scheme is introduced for estimation of PID sliding surface coefficients, due to which it reduces the reaching time as well as disturbance effect.The simulation results are presented to make a quantitative comparison with the traditional sliding mode control. It is demonstrated that the proposed control law improves the tracking performance of system dynamic model in case of external disturbances and parametric uncertainties.

KEYWORDS: *Second order sliding mode control, Big Bang- Big Crunch (BBBC) optimization, PD sliding surface, PID and Lyapunov stability.*

I. Introduction

Sliding Mode Control (SMC) is known to be a robust control method appropriate for controlling uncertain systems. High robustness is maintained against various kinds of uncertainties such as external disturbances and measurement error. The dynamic performance of the system under the SMC method can be shaped according to the system specification by an appropriate choice of switching function. Robustness is the most excellentbenefit of a SMC and systematic design procedures are well known in available literature [1-3].Even though, SMC is a powerful control method that can produce a robust closed-loop system under plant uncertaintiesand external disturbances, it suffers from lots of shortcomings. In idealSMC, the switching of control occurs at infinitely highfrequency to force the trajectories of a dynamic system to slide along therestricted sliding mode subspace. In practice, it is not possible to change the control infinitely fast because of the time delay for control computations andphysical limitations of switching devices. As a result, the SMC action can lead to high frequency oscillations called chattering whichmay excite unmodeled dynamics, energy loss, and system instability andsometimes it may lead to plant damage.Creation of a boundary layeraround the sliding manifold and the use of observers were attempted for chattering removal.However, the control still acts on the derivative of the sliding variable. Emel'yanov et al.[4] initially presented the idea of acting on the higher derivatives of the sliding variable andprovided second order sliding algorithms such as the twisting algorithm, and algorithm witha prescribed law of convergence [5]. Another so called real second order drift algorithmwas presented in Emel'yanov et al. [6]. The term real means that the algorithm has a meaningonly when there is a switching delay. In the ideal case of infinite frequency switching, itceases to converge. The so calledsuper twisting algorithm, which is applicable to systemswith relative degree one with respect to the sliding variable, [7] completely removes chattering.Levant [8] reported that in second order sliding, the sliding accuracy is proportionalto the square of the switching time delay which turns out to be another advantage of higherorder sliding modes.Since then, a number of such controllers have been developed in the literature and are currently finding useful applications.Bartolini et al. presented a sub-optimal second ordersliding algorithm in [9-11].

The higher order sliding mode concept was applied to thecontrol of constrained manipulators in Bartolini et al. [12]. Levant et al. [13] have applied itto aircraft pitch control and the same idea has been implemented for robust exact differentiationin [14]. Levant [15] demonstrated that the twisting algorithm with a variable integrationstep size performs better in the presence of measurement errors.Eker [16] proposed second order sliding mode control based on a PID sliding surface with independent gain coefficients and address issues related to SMC implementation on practical electromechanical plant. The second-order sliding mode correspondsto the control acting on the second derivative of the sliding surface[16]. It should be noted that different 2-SMC algorithmshave been presented in the literature such as the `twisting' and`super-twisting' [17], `sub-optimal' and `global' [18,19], asymptoticobserver algorithms [20], and drift algorithm [21]. Readers arereferred for the extensive information about these methods to[18, 21,22].The outline of this paper can be summarized as follows: Section 2 presented traditional sliding mode control for nonlinear system. In section 3, the design of second order sliding mode with PID sliding surface based on Big Bang- Big Crunch optimizationintroduced. Then, Section 4 describes the close loop stability analysis based on Lyapunov theory. MATLAB/Simulink based numerical simulations for stabilizing second order nonlinear system presented in Section 5. Finally, conclusions are given in Section 6.

II. First order Sliding Mode Control

Consider a second order nonlinear system, which can be represented by the following equation

$$\ddot{x} = f(x) + g(x)u + d(t) \tag{1}$$

where, $x = [x\,\dot{x}]^T \in R^n$ is the state vector with initial condition $x_0 = x(t_0)$, $f(x)$ and $g(x)$ are the nonlinear dynamic function, $u \in R^m$ is the control input vector and $d(t)$ is the external disturbance. The control problem is to synthesize a control law u such that state vector x traces the desired trajectory $x_d = [x_d\,\dot{x}_d]^T$ within the tolerance error bound defined by

$$\|x - x_d\| \le \gamma_1, \|\dot{x} - \dot{x}_d\| \le \gamma_2, \gamma_1 > 0, \gamma_2 > 0 \tag{2}$$

It is assumed that, $x_d(t)$, $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ are well defined and bounded for all time t. Tracking error is defined as $e = x - x_d$ and Here, sliding mode is said to be first order sliding mode if and only if S(t) = 0 and $S(t)\dot{S}(t) < 0$ is the fundamental sliding condition for sliding mode. The main objective of first order SMC is to force the state to move on the switching surface S(t) = 0. Let us consider the sliding surface $S(t)$ be expressed as in the state space R^2 by $S(x;t) = 0$

$$S = \left(\lambda + \frac{d}{dt}\right)^{n-1} e(t) \tag{3}$$

Where, n denotes order of uncontrolled system and λ is positive constant. The nonlinear system is having second order ie n=2 and then, in a two dimensional phase plane related to the sliding surface equation, the solution is a straight line passing through the origin, afterwards maintaining the error on sliding surface will result in e(t) approaching zero. Therefore, S(t) =0 is a stable sliding surface and e(t)→0 and t→∞. The derivative of sliding surface is given as follows,

$$\dot{S} = \lambda \dot{e}_{(t)} + \ddot{e}_{(t)} \tag{4}$$

The second time derivative of error can be written in terms of the plant parameters

$$\ddot{e}_{(t)} = \ddot{x} - \ddot{x}_d = f(x) + g(x)u - \ddot{x}_d \tag{5}$$

The above equation substituted in Eq. (4)

$$\dot{S} = \lambda \dot{e}_{(t)} + f(x) + g(x)u - \ddot{x}_d \tag{6}$$

The control input can be given as

$$u(t) = u_{eq}(t) + u_{sw}(t) \tag{7}$$

Where, $u_{eq}(t)$ is equivalent control and $u_{sw}(t)$ is switching control, respectively. The equivalent control is based on system parameters with disturbance assumed to zero. While switching control ensures the discontinuity of the control law across sliding surface, providing additional control to account for the presence of matched disturbances and unmodeled dynamics.

The equivalent control is derived from sliding condition, When Eq.(6) is become zero, which is the necessary condition for the tracking error to remain on the sliding surface.

$$u_{eq}(t)=\frac{-1}{g(x)}\left(\lambda\dot{e}_{(t)}+f(x)-\ddot{x}_d\right) \quad (8)$$

The switching input is introduced as follows

$$u_{sw}(t)=k_d\,\mathrm{sgn}(S(t)) \quad (9)$$

Where, k_d is a positive constant and sign(.) denotes sign function defined as follows

$$sign(s)=\begin{cases}1 & s>0\\ 0 & s=0\\ -1 & s<0\end{cases} \quad (10)$$

The signum function in the hitting control Eq.(9) requires infinite switching on the part of the control signal and actuator to maintain the system dynamics on the sliding surface. In practice, actuator limitations, transport delays, computational delays and other factors prevent true sliding leads to phenomenon known as chattering effect in which the states jump back and forth across the sliding manifold while decaying towards the origin. This effect can be eliminated by using second order sliding mode control technique.

III. Second Order Sliding Mode Control Optimization by Using BBBC

In the higher order sliding mode controller category, the second order sliding mode controller (SOSMC) has been extensively developed. This is compared to the first order sliding mode control has advantage of lesser chattering with better convergence accuracy. The main idea of SOSMC is not only to force the sliding surface but also its second order derivatives to zero.

Let us consider PID sliding surface with constant coefficients expressed as

$$\dot{S}(t)+\beta S(t)=K_P e(t)+K_I\int_0^t e(t)\,dt+K_D\dot{e}(t) \quad (11)$$

Where, $e(t)$ is the tracking error, $\dot{e}(t)$ is the time derivative of $e(t)$, x_d the desired trajectory. K_P, K_I and K_D are designed such that the sliding mode on $S=0$ is stable ie the convergence of S to zero in turn guarantees that $\tilde{x}$ and $\dot{\tilde{x}}$ also converge to zero. The coefficients of PID based sliding surface are strictly positive constant K_P, K_I and $K_D \in R^T$. Taking the derivative of sliding surface with respective time, defined by Eq.(11) can be given as

$$\ddot{S}(t)+\beta\dot{S}(t)=K_P\dot{e}(t)+K_I e(t)+K_D\ddot{e}(t) \quad (12)$$

A necessary condition for the output trajectory to remain on the sliding surface $S(t)$ is $\ddot{S}(t)=0$

$$K_P\dot{e}(t)+K_I e(t)+K_D\ddot{e}(t)=0 \quad (13)$$

The control effort being derived as the solution of $\ddot{S}(t)=0$ without considering lumped uncertainty $L(t)=0$ is to achieve the desired performances under the nominal model and it is referred to as equivalent effort represented by u_{eq}

$$u_{eq}(t)=-(K_D g(x))^{-1}\left[K_D(f(x)+d(t)-\ddot{x}_d)+K_P\dot{e}(t)+K_I e(t)\right] \quad (14)$$

If unpredictable perturbations from the parameter variations or external load disturbance occur, the equivalent control effort can't ensure the favorable control performance. Thus the auxiliary control effort should be designed to eliminate the effect of unpredictable perturbations. The auxiliary control effort is referred to as hitting control effort represented by u_{dis}.

In SOSMC, u_{dis} is given as follows

$$u_{dis}=\alpha S(t)+K_D\,\mathrm{sgn}(\dot{S}(t)) \quad (15)$$

Where, K_D is a hitting control gain concerned with the upper bound of uncertainties and sgn(.) is a sign function.The controller must be designed such that it can drive the output trajectory to the sliding mode $S(t)=0$. The output trajectory, under the condition that it will move towards and reach the sliding surface, is said to be on reaching phase.

Estimation of hitting gain and sliding surface coefficients:

In the proposed controller the sliding surface differential is used and K_D is calculated with an adaptive equation. Using the proposed controller the results caused more adequate effect on the smoothness of the control input, compared with sliding mode control in the initial time.It is assumethe modelingerroris boundedandtheupper boundis $d_0 + d_1\|e(t)\|$, where d0, d1 are positive constant and disturbance isbounded $\|d\| < \beta$. Defining a new parameter θ as given

$$\theta_i = \beta + d_0 + d_1\|e(t)\| \qquad (16)$$

The discontinuous gain can be expressed as

$$K_D = \hat{\theta}_i + \mu_i \qquad (17)$$

Where, μ_i is positive constant and adaptive control is given by [23]

$$\dot{\hat{\theta}}_i = f\|\tilde{e}\|\|\gamma_i\| \qquad (18)$$

Here, $\gamma = [\dot{s}, \ddot{s}]$ and *f*is positive constant used for adaptation of discontinuous gain in SOSMC.

In this control algorithm sliding surface coefficients K_P, K_I and K_D are are determined by using BBBC optimization algorithm.Here proportional gain is assumed to be unity, But integral gain and derivative gain are estimated. The BBBC method was developed and proposed as a novel optimization method by Erol and Eksin (2006) This method is derived from one of the evolution of the universe theories in physics and astronomy, describing how the universe was created, evolved and would end, namely the BBBC phase. The big bang phase comprising random energy dissipation over the entire search space or the transformation from an ordered state to a disordered or chaotic state. After the big bang phase, a contraction occurs in the big crunch phase. Here, the particles that are randomly distributed are drawn into an order. This aims to reduce the computational time and have quick convergence even in long, narrow parabolic shaped flat valleys or in the existence of several local minima.

The big bang phase is somewhat similar to creation of initial random population in GA. The designer should handle the impermissible candidates at this phase. Once the population is created, fitness values of the individuals are calculated (Genç, Eksin, & Osman, 2010). The crunching phase is a convergence operator that has many inputs but only one output, which can be referred to as the center of "mass." The center of mass represents the weighted average of the candidate solution positions. Here, the term mass refers to the inverse of the merit function value (Singh &Verma, 2011). After a number of sequential banging and crunching phases the algorithm converges to a solution. The point representing the center of mass "Xc" of the population is calculated according to the formula

$$X_C = \frac{\sum_{k=0}^{N-1} \frac{Xk}{f_k}}{\sum_{k=0}^{N-1} \frac{1}{f_k}} \qquad (19)$$

whereX_k is a point within an *n*-dimensional search space generated, here it is related to the numerator polynomial coefficients, f_k is a fitness function or objective value of the candidate *k* and *N* is the population size in banging phase. The convergence operator in the crunching phase is different from wild selection since the output term may contain additional information (new candidate or member having different parameters than others) than the participating ones. In the next cycle of the big bang phase, new solutions are created by using the previous knowledge (center of mass), the fitness function f_k (Erol&Eksin, 2006) is

$$f_k = \sum_{i=0}^{M-1}[y(i\Delta t) - y_r(i\Delta t)]^2 \; ; \quad M = \frac{T}{\Delta t} \qquad (20)$$

Where, y(iΔ t) and yr(iΔ t) are the unit step responses of the higher-order and the reduced-order models at time t=Δ t. Usually time T is taken as 10 s and Δ t=0.1 s.

The basic BBBC algorithm utilized here is as follows:

1. The Big bang starts by generating the new population. [Start

2. For each iteration the algorithm will act such [Evaluate Fitness value that each candidates will move in a direction to improve its fitness function. The action involves movement updating of individuals and evaluating the fitness function for the new position.

3. Compare the fitness function of the new [Compare Fitness Function position with the specified fitness function. Repeat the above steps for the whole set of candidates.

4. Stop and return the best solution if maximum [Maximum iteration iteration is reached or a specified termination

criterion is met. Else, update and start generating new population at step 1.

5. Go to step 2 for fitness evaluation. [Loop

IV. Numerical Simulation and Experimental Results

Speed control of DC Motor is chosen as an example to demonstrate the effectiveness of the proposed control algorithm as SOSMC with BBBC optimization. A simple model of a DC motor driving an inertial load shows the angular rate of the load, ω(t), as the output and applied voltage, V_{app}, as the input. The ultimate goal of this example is to control the angular rate by varying the applied voltage. Fig. 1 shows a simple model of the DC motor driving an inertial load *J*.In this model, the dynamics of the motor itself are idealized; for instance, the magnetic field is assumed to be constant. The resistance of the circuit is denoted by *R* and the self-inductance of the armature by *L*. The important thing here is that with this simple model and basic laws of physics, it is possible to develop differential equations that describe the behavior of this electromechanical system. In this example, the relationships between electric potential and mechanical force are Faraday's law of induction and Ampere's law for the force on a conductor moving through a magnetic field.A set of two differential equations given in (21) describe the behavior of the motor. The first for the induced current, and the second for the angular rate,

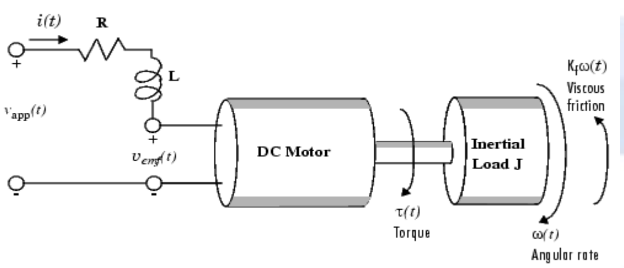

Fig. 1 DC motor driving inertial load

$$\frac{di}{dt} = -\frac{R}{L}\cdot i(t) - \frac{K_b}{L}\cdot \omega(t) + \frac{1}{L}\cdot V_{app}$$

$$\frac{d\omega}{dt} = -\frac{K_F}{J}\omega(t) + \frac{K_m}{J}\cdot i(t) \qquad (23)$$

Above state equation can be written in a second order uncertain form

$$\ddot{\omega}_l(t) = -(A_n \pm \Delta A)\dot{\omega}_l(t) - (B_n \pm \Delta B)\omega_l(t) + (C_n \pm \Delta C)v_a(t) + d \qquad (24)$$

Where, $\ddot{\omega}_l(t)$is the output, $\omega_l(t) \epsilon R$, u(t) is the control input and A_n, B_n, C_n are the nominal plant parameters, $\Delta A, \Delta B, \Delta C$ are the unknown model uncertainties.

$$\ddot{\omega}_l(t) = -(A_n)\dot{\omega}_l(t) - (B_n)\omega_l(t) + (C_n)v_a(t) + L(t,u) \qquad (25)$$

Where $L(t,u(t))$ denotes the lumped uncertainties.

In this simulation the nominal parameter values are calculated as $A_n = 118.1663, B_n = 783.5762, C_n = 644.0997$ with step input signal (armature voltage) with an amplitude of 4.5 V and output is the speed of DC motor in rpm.As observed in Fig. 2 even in the presence of external disturbances, the proposed control strategy is able to provide better trajectory response compared with conventional SMC. For comparative analysis, the phase portrait obtained with SOSMC based on PID sliding surface is presented in Fig. 3. Chattering problem was eliminated in SOSMC method, because of hyperbolic function of sliding surface and differentiator. The improved performance of SOSMC with BBBC optimization for sliding hyperplane over the traditional SMC as shown in Fig. 4, which is due to the ability of recognize and compensate the external disturbances with BBBC tuning approach to determine sliding coefficients.

Despite the external disturbance forces and uncertainties with respect to model parameters, the proposed control technique allows the DC motor to track the desired speed trajectory with a small tracking error as presented in Fig.5. In the above simulation, the parameters of the controllers were chosen based on the assumption that exact values of plant parameters are not known, but with a maximal uncertainty of ± 10% over the previous adopted values. For the dynamic model simulation of electromechanical plant, parameter values were selected as mentioned above. From the observation of simulated results, it was clear that SOSMC based PID sliding surface with BBC tuning approach was employed for avoiding oscillation in the output response when load disturbance force was applied and also reduce the reaching time.

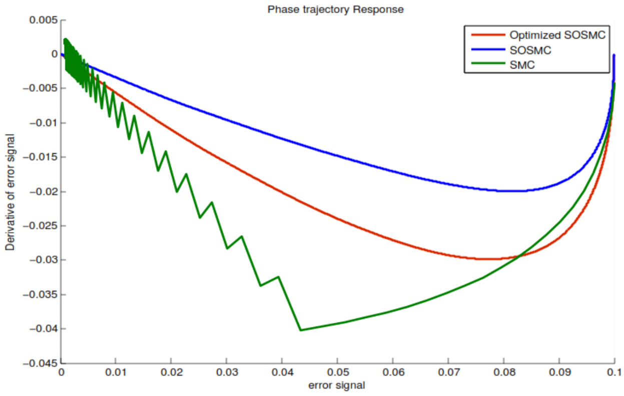

Fig. 3 Phase trajectory response of DC Motor

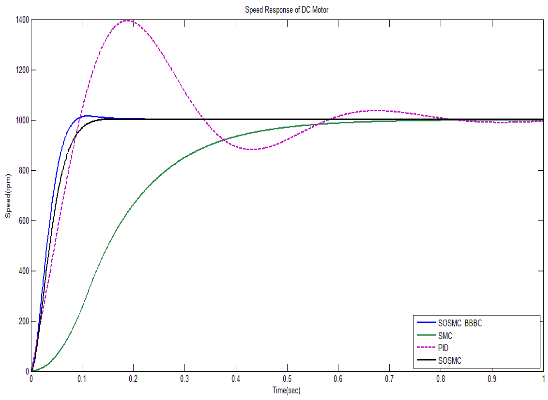

Fig.4 Set point tracking response of DC motor

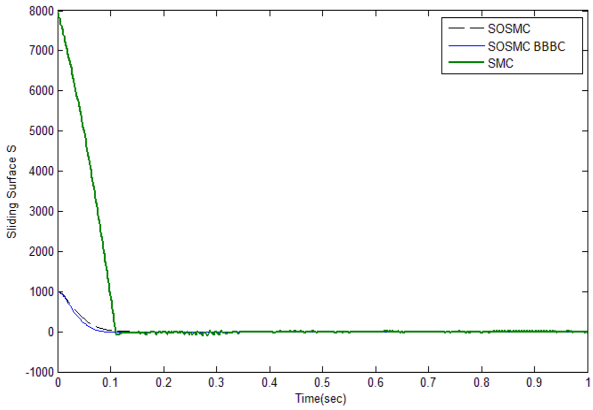

Fig.5 sliding surface response of DC motor

V. Conclusion

In this paper, SOSMC with PID sliding surface was employed to deal with the speed regulation of DC motor. An integral term expressed in the PD-type sliding surface that resulted in a type of PID sliding surface. Due to the integral term chattering problem was not appear in the phase trajectory response. In proposed control algorithm, hitting gain is obtained from adaptive law along with BBBC optimization technique utilized to determine PID sliding surface coefficients. The disturbance effect was identified and compensated by using optimized SOSMC technique. The stability and convergence properties of closed loop systems were analyzed through phase trajectory response. Through numerical simulations, the improved performance of proposed control technique over the conventional SMC was demonstrated.

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