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Note on Co Ideals in Ternary Semigroups

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ABSTRACT

In this paper we study properties of co-ideal in the ternary semigroup $< z_0^+, . >$

KEYWORDS: *ternary semigroup, ternary semiring, co-ideal, subtractive co-ideal.*

I. Introduction

Cayley and Sylvester along with several other mathematicians, in the 19th century considered ternary algebraic structures and cubic relations. The nary structures which are the generalization of ternary structures create hopes because of theis possible applications in physics. A few important physical applications have been recorded in ternary semigroups exhibit natural examples of ternary algebras. Also S. Kar [7] studied the ideal theory in the ternary semiring $< z_0^+, . >$.

In this paper , we introduce the concept of a co-ideal and study properties of a ternary semigroups $< Z_0^+, . >$

II. Preliminaries

Definition 1.1: A non- empty set T together with a ternary operation called ternary multiplication, denoted by juxtaposition is said to be a ternary semigroup if $(abc)de = a(bcd)e = ab(cde)$ for $a, b, c, d, e \in T$.

Definition 1.2: An element e of a ternary semigroup T is called

1. a right identity (or right unital element) if $xee = x$ for all $x \in T$.
2. a left identity (or left unital element) if $eex = x$ for all $x \in T$.
3. a lateral identity (or lateral unital element) if $exe = x$ for all $x \in T$.
4. a two-sided identity (or bi-unital element) if $eex = xee = x$ for all $x \in T$.
5. an identity (or unital element) if $eex = exe = xee = x$ for all $x \in T$.

Example 1.3: Let $< Z_0^+, . >$ be the set of all positive integers. Then with the usual ternary multiplication. $< Z_0^+, . >$ forms a ternary semigroup with zero element 0 and identity element 1.

Definition 1.4: A non-empty subset S of a ternary semigroup T is called a ternary subsemigroup if $s_1s_2s_3 \in T$ for all $s_1, s_2, s_3 \in S$.

Definition 1.5: A ternary subsemigroup I of a ternary semigroup T is called

1. a right ideal of T if $ITT \subseteq I$
2. a left ideal of T if $TTI \subseteq I$
3. a lateral ideal of T if $TIT \subseteq I$
4. a two-sided ideal of T if I is both left and right ideal of T.

5. An ideal of T if I is a left, right and lateral ideal of T.

An ideal I of a ternary semigroup T is called a proper ideal if I ≠T.

2. Co-ideals in ternary semigroup.

Definition 2.1: A non-empty subset I of a ternary semigroup T is called a co-ideal if

- $a, b, c \in$ I implies $abc \in$ I
- $a \in$ I, $t \in$T implies $at \in I$

Definition 2.2: A co-ideal I of a ternary semigroup T is called subtractive if $a, b, abc \in$ I, $c \in$ T, then $c \in$ I.

Theorem 2.3: Let I, J be a co-ideal of a ternary semigroup T. Then

(1) $I \cap J$ is a co-ideal of T,

(2) IJ is a co-ideal of T.

(3) $IJ \subseteq I \cap J$.

Proof : (1) Let $a, b, c \in I \cap J$ then $abc \in I$, since I, J is a co-ideal and so $abc \in J$. Hence $abc \in I \cap J$. Let $a \in I \cap J$ and $r \in T$ then $at \in I$ and $at \in J$, since I, J is a co-ideal of T. Thus $at \in I \cap J$. Hence $I \cap J$ is a co-ideal of T.

(2) Let $x = ab$, $y = a'b'$, $z = a^*b^* \in IJ$. Then

$xyz = (ab)(a'b')(a^*b^*)$

$= [(aa')(ab')(ba')(bb')](a^*b^*)$

$= ([(aa')(a^*b^*)][(ab')(a^*b^*)] [(ba')(a^*b^*)] [(bb')(a^*b^*)]$

$= [(a a^*)(a b^*)(a'a^*)(a'b^*)] [(a a^*)(a b^*)(b'a^*)(b'b^*)] [(b a^*)(b b^*)(a'a^*)(a'b^*)]$
$[(b a^*)(b b^*)(b'a^*)(b'b^*)]$

$= [(a a^*)(a b^*)(a'a^*)(a'b^*)(a a^*)(a b^*)(b'a^*)(b'b^*)(b a^*)(b b^*)(a'a^*)(a'b^*)$
$(b a^*)(b b^*)(b'a^*)(b'b^*)]$

$= (a a^*)^2 (a b^*)^2 a'(a^*b^*) b'(a^*b^*) b(a^*b^*) a' (a^*b^*)b (a^*b^*) b' (a^*b^*)$

$= (a a^*)^2 (a b^*)^2 a' b' b a' b b' (a^*b^*)$

$= (a a^*)^2 (a b^*)^2 a'^2 b^2 b'^2 (a^*b^*)$

Since $(a^*b^*) \in IJ$ we have $xyz \in IJ$. Let $r \in T$ then $xr = (ab)r \in I$, since I is a co-ideal of T. Hence IJ is a co-ideal of T.

(3)Let $x = ab \in IJ$ where $a \in I, b \in J$. Now $a \in I, b \in T$ and I is a co-ideal implies $x = ab \in I$. Similarly $x \in J$. Hence $x \in I \cap J$ so $IJ \subseteq I \cap J$.

Corollary 2.4: Every ideal is a co-ideal.

Proof : Let I is an ideal of ternary semigroup of T and $a, b, c \in I$. Then $abc \in I$. Let $a \in I$ and $t \in T$ then $at \in I$, since I is an ideal of T. Thus I is a co-ideal of T.

Theorem 2.5: Let I, J be a subtractive co-ideal of a ternary semigroup T. Then $I \cap J$ is a subtractive co-ideal of T.

Proof: By theorem (2.3(1)) we have $I \cap J$ is a co –ideal of T. If $a, b, abc \in I \cap J$, $c \in T$ then $c \in I \cap J$, since I, *J is a subtractive co –ideal of T*.

Lemma 2.6: Let I, J be a subtractive co-ideal of a ternary semigroup T and $a \in I$, $b \in J$, $c \in I \cap J$. Then the following conditions are equivalent

(1) $abc \in I \cap J$,

(2) $a \in I \cap J$ or $b \in I \cap J$,

(3) $a, b \in I$ or $a, b \in J$

Proof : (1)⟹(2) Let $abc \in I \cap J$. Without loss of generality assume that $abc \in I$. Since I is a subtractive co-ideal we have $b \in I$. Hence $b \in I \cap J$.

(2) ⟹(3) Let $a \in I \cap J$ or b $\in I \cap J$. We must to show that $a, b \in I$ or $a, b \in J$

Case 1: If $a \in I \cap J$ then $a \in I$ and $a \in J$.

Case 2: If $b \in I \cap J$ then $b \in I$ and $b \in J$

From case1.,2. We have $a, b \in I$ or $a, b \in J$.

(3)⟹ (1) Let $a, b \in I$ or $a, b \in J$ then $ab \in I$ or $ab \in J$. Since I, J is a subtractive co-ideal and $c \in I \cap J$ we have $abc \in I \cap J$.

3. Co-ideals in Ternary Semigroup $< Z_0^+, . >$

Lemma 3.1: $I_n = \{ a \in Z_0^+ : a \leq n \}$ is a co-ideal.

Proof : Let $a, b, c \in I_n$ then $a, b, c \in Z_0^+$. Thus $abc \in Z_0^+$ so $abc \in I_n$. Let $a \in I_n$ and $r \in T$ then $ar \in Z_0^+$. Thus $abc \in I_n$. Hence I_n is a co-ideal of Z_0^+.

Theorem 3.2: A non-empty subset I of the ternary semigroup $< Z_0^+, . >$ is a co-ideal if and only if $I = I_n$

Proof : Assume that I is a co-ideal of $< Z_0^+, . >$. We must show that $I = I_n$ Since I is a non-empty, I has the largest element say n. Let $x \in I_n$ then $x \leq n$ and $x = ny$ for some $y \in Z^+$ Now $x \in I$, since I is a co-ideal. Hence $I_n \subseteq I$. But $I \in I_n$. So $I = I_n$. Assume that $I = I_n$ lemma 3.1 then I_n is a co-ideal of Z_0^+. Thus I is a co-ideal of $< Z_0^+, . >$

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