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Study of Composite and Prime Labelling

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ABSTRACT

In this paper, we introduce the concept of composite labelling and prime labeling of bipartile, complete bipartile, complete, regular,spanning subgraph, isomorphic graph and underlying subgraph.A graph that admits compositelabelling is known as a composite graph and admits prime labelling is known as a prime graph. We investigate composite labelling and prime labellingfor bipartile, complete bipartile, complete, regular,spanning subgraph isomorphic graph and underlying subgraph

Keywords: *Graph Labelling, Composite Labelling, Prime Labelling, Underlying subgraph, Composite graph, Prime graph*

I. INTRODUCTION

We consider undirected connected simple graph G=(V, E) with vertex set V(G) and edge set E(G). Let |V(G)| be denoted by n and |E(G)| be denoted by m, known as the order and size of G respectively.

All the graphs considered are undirected connected simple graphs with order n and size m. Let$u$, v, $w \in$ V(G). A composite labelling is a bijective function f: (V(G) ∪ E(G))→ {1, 2, 3, ...,m+n} such that greatest common divisor (f(uv),f(vw))≠1 and also prime labelling is a bijective function f:(V(G) ∪ E(G))→ {1, 2, 3, ...,m+n} such that greatest common divisor (f(uv),f(vw))=1

Definition 1.1. Graph labelling is an assignment of labells to vertices or edges or to bothsubject to certain conditions.

The concept of prime labelling originated with Entringer and it was later introduced by Tout, Dabbousy and Howalla.[3] They defined prime labelingfor a graph G with vertex set V, as a bijective function f: V(G) ! {1, 2, ..., p} where p is the number of vertices such that each vertex receives a distinct integer from {1, 2, ..., p} and for each edge $xy$the labels assigned to x and y are relatively prime.

Vaidya and Prajapathy[6] introduced the concept of k-prime labelling. Ramasubramanian and Kala[8] introduced the concept of total prime graph. They defined total prime labelling as a bijective function f: V(G)[E(G) ! {1, 2, 3, ...,m + n} such that for each edge uv, the labels assigned to vertices u and v are relatively prime and for each vertex the greatest common divisor of the labels on the incident edges is 1and also each vertex uv, the labels assigned to edges u and v are relatively prime and for each edge the greatest common divisor of the labels on the incident vertex is 1.

In this paper, we investigate composite labelling as well as prime labelling. We identify a few families of graphs which admit composite labelling and prime labelling.

Definition 1.2. Let G=(V, E) be an undirected connected simple graph of order n and size m. Let $u, v, w \in$ V(G). A composite labelling is a bijective function f: V(G)∪E(G)→ {1, 2, 3, …,m + n} such that gcd (f(uv), f(vw)) ≠ 1.

It can easily be seen that the labelling introduced here is a variation of total labelling. However, in the composite labelling, the stress is given to the edge labelling even though labels of vertices are crucial in labelling the edges.However, in the prime labelling, the stress is given to the edge labelling even thoughlabels of vertices are crucial in labelling the edges.

Definition 1.3. A graph that admits composite labelling is known as a composite graph.

It would be interesting to obtain a bound for the value of $m + n$ so that the graph admits composite labelling.

Definition 1.4. A graph that admits prime labelling is known as a prime graph.

It would be interesting to obtain a bound for the value of $m + n$ so that the graph admits prime labelling.

Theorem 1.5. For any simple connected undirected graph with n vertices m edges admitting composite labelling and prime labelling we have, 2n-1≤ m+n $\leq \frac{n(n+1)}{n}$

Proof: In order to obtain the upper bound, let us consider a complete graph whose number of edges is given by ${}^{n}c_2$. Therefore,m+n $\leq \frac{n(n+1)}{n}$.

For obtaining the lower bound, we consider a tree with n vertices and $n - 1$ edges. We get 2n-1≤ m+n.Therefore we have, 2n-1≤ m+n $\leq \frac{n(n+1)}{n}$

Theorem 1.5. All trees admit composite labelling and prime labelling.

Proof: Trees are acyclic connected graphs with n vertices and $n - 1$ edges. We have to consider the following cases.

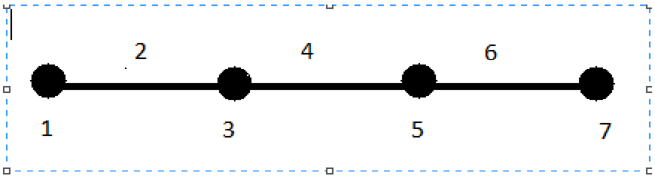
(i) **Composite labeling**

Case 1. n is even.

When n is even, then $n - 1$ is odd. Therefore $m + n = 2n - 1$, which is odd. We can label the vertices of a tree with odd labels and edges with even labels so that composite labelling is obtained by trees.

Case 2. n is odd.

When n is odd, $n - 1$ is even. The proof is similar to that of **Case 1**.



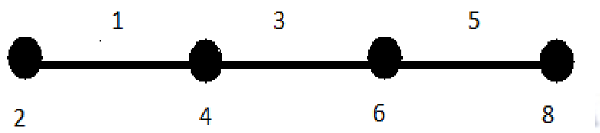
(ii) **Prime labeling**

Case 3. n is even.

When n is even, then $n - 1$ is odd. Therefore $m + n = 2n - 1$, which is odd. We canlabel the vertices of a tree with even labels and edges with odd labels so that primelabelling is obtained by trees.

Case 4. n is odd.

When n is odd, n - 1 is even. The proof is similar to that of **Case 3.**



II. MAIN RESULT

In this section, we discuss the composite and prime labelling of undirected simple connected graph and underlying simple subgraph.

Theorem 1.6. All undirected simple graph satifies the composite labeling

Proof: Let n denote the number of vertices and m denote the number of edges. Let the undirected simple graph admits composite labelling .Hence it does not contain loops and parallel edge so degree of the vertex is regular.

The following undirected simple subgraph with composite labelling.

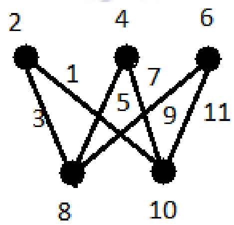


fig.1 Bipartile

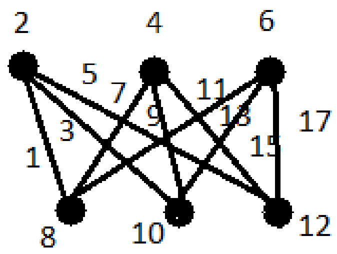


fig.2 Complete bipartile

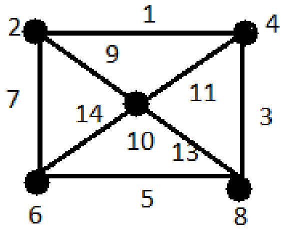


fig.3 Complete graph

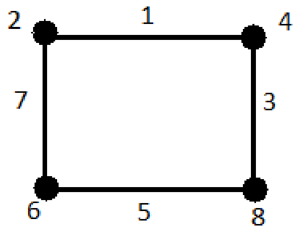


fig.4 Regular graph

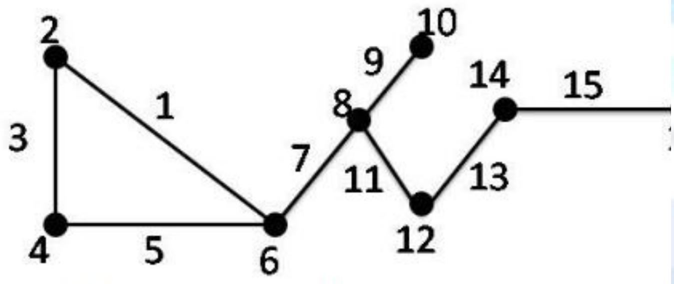


fig.5 Spanning subgraph

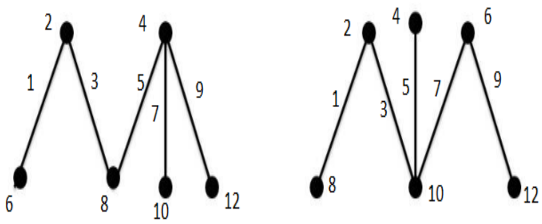


Fig.6 Isomorphic graph

Theorem 1.7. All undirected simple graph satifies the prime labelling

Proof: Let n denote the number of vertices and m denote the number of edges. Let the undirected simple graph admits prime labelling .Hence it does not contain loops and parallel edge so degree of the vertex is regular.

The following undirected simple subgraph with prime labelling.

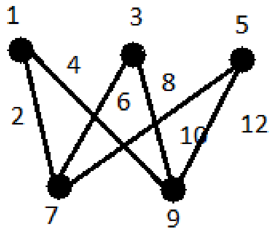


fig.7 Bipartile

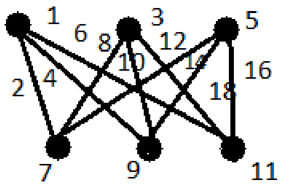


fig.8 Complete bipartile

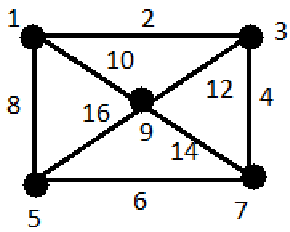


fig.9 Complete graph

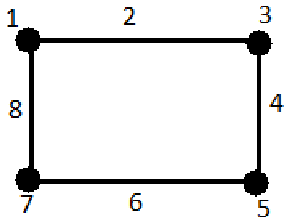


fig.10 Regular graph

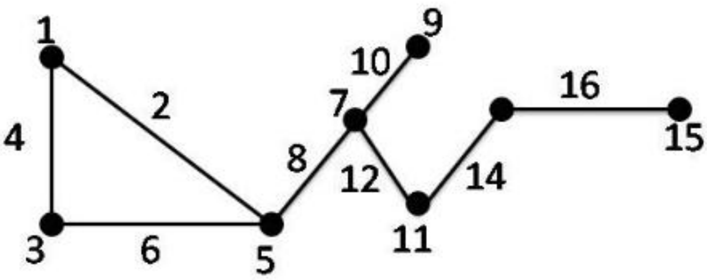


fig.11 Spanning subgraph

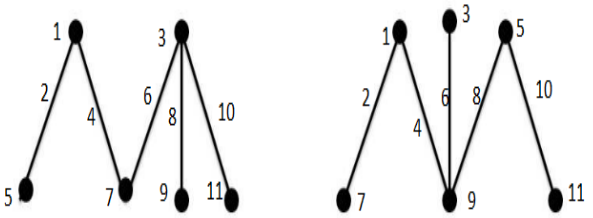


Fig.12 Isomorphic graph

Theorem 1.8. A composite and prime labelling are not satisfied in all underlying subgraph

Proof: Let n denote the number of vertices and m denote the number of edges. Let the underlying simple graph G not satisfied the composite labelling and prime labeling, because it would contain the parallel edges and loops it contains the vertices $v \geq 2$. So the graph does not satisfy the composite and prime labeling

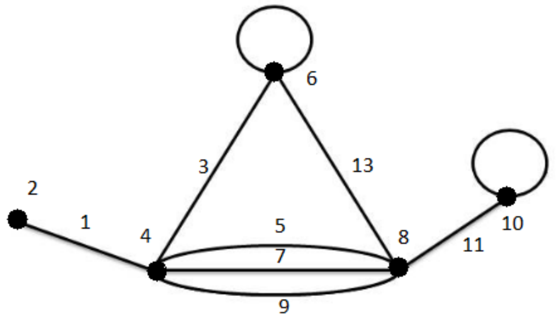


Fig.13 Underlying subgraph G (composite labeling)

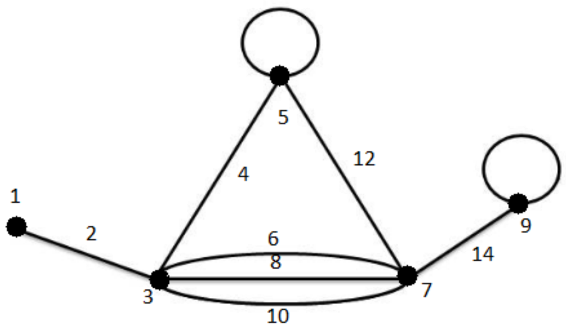


Fig.14 Ungerlying subgraph G (Prime labelling)

III. Conclusion

Finally, we conclude that undirected simple graph attain composite and prime labelling but underlying subgraph not obtain composite and prime labelling in the given condition . When vertices of larger degrees exist in a graph, it is difficult to find the composite labelling and prime labelling.

Hence, provide a larger scope for strong research in the area of graph theory.

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