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Estimation of Parameters of U-Quadratic Distribution by using Weighted Least Squares (WLS)

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ABSTRACT

The purpose of this study is to estimate parameters and create an optimal grouped sample without any prior knowledge or guess values of parameters. For this, we use the Weighted Least Squares Estimator Method to determine the parameters of the U-quadratic Distribution. We compare equispaced and antipodal optimally constructed grouped data using the Asymptotically Relative Efficiency method. We also evaluate the performance of the estimators by computing various statistical metrics such as Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE), Relative Absolute Bias (RAB), and Relative Error (RE) for both parameters based on 1000 simulations under grouped sample.

Keywords: Weighted Least Square, U-quadratic Distribution, equispaced and antipodalOptimal, Grouped sample

1. INTRODUCTION

In order to determine the unknown parameters of the probability density function for each measurement, maximum likelihood estimation is used to maximize a function. This is achieved by either maximizing the logarithm of the likelihood function or minimizing the weighted sum of squares of the residuals. The weighted least squares estimator is the solution to this problem, as proposed by Abur et al. in 2016. By incorporating the measurement error covariance matrix, the state estimator becomes a weighted least squares estimator, which accurately weighs the importance of each measurement. Nithin V G and Libish T M proposed the Smart Grid State Estimation by Weighted Least Square Estimation in 2016. However, some previous studies suggest that the classical WLS method may encounter difficulties when there are errors in the measurements.

Taking into account the challenges posed by bad measurements on the estimated results, this work proposes a new approach
to the Weighted Least Squares (WLS) method. By using a non-linear programming model, the proposed strategy is aimed at reducing the impact of bad measurements, thus helping to address the research gap on the effectiveness of the WLS method in solving the state estimation problem. The proposed approach enables the inclusion

of new variables and operational constraints that cannot be accommodated in the traditional WLS method. Furthermore, representing the state estimation as a mathematical optimization problem has the advantage of easy implementation and solution using widely-used mathematical optimization software in U-Quadratic distribution studies and research.

We will be discussing the procedure to estimate unknown parameters for the UQuadratic distribution. Weighted Least Squares (WLS) is a popular statistical inference method for this purpose due to its desirable properties such as consistency, asymptotic efficiency and invariance. In this paper, we present a Monte Carlo simulation procedure to simulate data for WLS of the unknown parameters of the U-quadratic distribution. We also compute various performance metrics, including Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), and Relative Error (RE), for both the parameters using 10,000 simulations. Additionally, we derive asymptotic confidence bounds s for the unknown parameters. To illustrate the methodology, we apply it to two important datasets and show that the U-quadratic distribution is a simple alternative to be used for lifetime data analysis.

A random variable, say, X, is said to follow a U-Quadratic distribution if its DF (distribution function) Q (x; £,ŋ) is given by

Q (x; £,ŋ) = £3 (x – ŋ) 3 + (ŋ – £)3 … (1)

A random variable, say X, is said to follow a U-Quadratic distribution if its PDF (Probability distribution function) q(x; £, ŋ) is given by

q(x; £, ŋ) = £ (x – ŋ)2 … (2)

Here, £ = 12(u – i)3 ; ŋ = u+l2 ; x ∈ [l, u]

A random variable X ~ U-Quadratic distribution with parameters (£,ŋ) has a Quantile function and is in the form The pth quantile xp of U-Quadratic distribution is the root of the equation.

xp = 3p£- (ŋ – £)313 + ŋ …(3)

Let U~ U(0,1), then equation (3) can be used to simulate a random sample of size n from the U-Quadratic distribution as follows.

Xi = 3Ui£- (ŋ – £)313 + ŋ …(4)

2. ESTIMATION OF PARAMETERS OF U-QUADRATIC DISTRIBUTION USING THE METHOD OF WEIGHTED LEAST SQUARES

Assume that X(1) ≤X(2)≤X(3)≤…≤X(n) are the order statistics probability sample from any probability distribution. The ith order statics has the mean and variance as follows:

E (F(Xi)) = F(i) = in+1 and var(F(Xi))

= i (n-i+1)(n+1)2(n+2) … (5)

Weighted Least- Squares proposed by Swain et al. (1988). We obtain the parameters of U-Quadratic distribution by using WLS to minimise the function with respect to (£, ŋ) parameters, as follows:

W = i=1nWi((F(Xi)-E (F(Xi))) 2 … (6)

Wi = (n+1)2(n+2)i(n-i+1)

Where F(Xi) represents the theoretical cumulative distribution function of the observation Xi of the distribution under study and E (FXi) = F(i) represents the empirical cumulative distribution function which is usually estimated by F(i) = in+1; then, we obtain the following

O = i=1nWi((F(Xi)-in+1) 2 … (7)

O(x; £, ŋ) = i1n(n+1)2(n+2)i(n-i+1)£3 (x(i) – ŋ)3 + (ŋ – £)3-in+1 2 …(8)

We can determine the WLS estimates by minimizing (8) with respect to the parameters via the function minimizing method, and we get the following equations

dO(x; £, ŋ)d£ = dd£i=1n(n+1)2(n+2)i(n-i+1)£3 (x(i) – ŋ)3 + (ŋ – £)3-in+1 2 = 0

=> dO(x; £, ŋ)d£=i=1n(n+1)2(n+2)i(n-i+1)£ (xi – ŋ)2- 3(ŋ – £)2] = 0 … (9)

dO(x; £, ŋ)dŋ = ddŋi=1n(n+1)2(n+2)i(n-i+1)£3 (x(i) – ŋ)3 + (ŋ – £)3-in+1 2 = 0

=> i=1n(n+1)2(n+2)i(n-i+1)-£ (xi – ŋ)2+ 3(ŋ – £)2] = 0

… (10)

dO(x; £, ŋ)dŋd£ = ddŋi=1n(n+1)2(n+2)i(n-i+1)£ (xi – ŋ)2-

3(ŋ – £)2] = i=1n(n+1)2(n+2)i(n-i+1)(-2£ (xi – ŋ) + 6 (ŋ – £))

…(11)

3. ASYMPTOTIC CONFIDENCE BOUND

In this context, we are deriving the asymptotic confidence bounds for unknown parameters: Scale (£) and Shape (ŋ), given that £> 0 and ŋ> 0. The simplest approach for a large sample is to assume that the Weighted Least Squares (WLS) (£,ŋ) are approximately normal, with a mean of (£,ŋ) and covariance matrix I0-1. Here, I0-1 denotes the inverse of the observed information matrix, which is defined as follows:

I0-1=-E(d2lnd£2)-E(d2lnd£dŋ)-E(d2lnd£dŋ)-E(d2lndŋ2)

The Asymptotic (1-r)100% Confident t intervals for estimated parameters are as follows

£+zr2var£,

ŋ+zr2varŋ

4. SIMULATION STUDY

We will now discuss the simulation study that we conducted. The primary objective of these simulations was to determine the efficiency of the Method of Weighted Least Squares estimation method for the parameters of the UQuadratic distribution. Here's how we went about the procedure:

Step 1: Set the sample size 'n' and the vector of parameter values Φ=(£, ŋ).

Step 2: Using the values obtained in step 1

step (2), compute £OLS and ŋOLS through the Method of Weighted Least Squares.

Step3: Repeat steps (2) and (3) N times

Step4: Using Φ of Φ, compute the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If Φlm is Method of Weighted Least Squares estimate method of Φm, m=1, 2 where Φmis a general notation that can be replaced by Φ1=£, Φ2=ŋ based on sample l, (l =1,2,…,r), then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

Average Estimate (Φm)= i=1rΦlmr

Variance(Φm) = i=1r(Φlm- Φlm)2r

SD (Φm)= i=1r(Φlm- Φlm)2r

Mean Absolute Deviation(Φm) = i=1rMed(Φlm- Φlm)r

MeanSquareError (Φm) = i=1r(Φlm-Φm)2r

RelativeAbsoluteBias(Φm) =i=1r(Φlm-Φm)rψm

RelativeError(Φm)= 1ri=1rMSE(Φlm)Φm2

The results were computed using the software R (R Core Development Team). The seed is used to generate the random values. The chosen values to perform this procedure were N = 10,000 and n = (10, 30, 60,80,150 and 250) for different population parameter values Φ=(£, ŋ).

5. APPLICATIONS

In this section, we analyzed two real datasets. The dataset contains 30 observations of the driving distance of hitting a golf ball, which records the most hits of a golf ball on a golf club in 30 seconds. The second dataset is related to the ages (in months) of 10 patients who died from COVID-19. Our simulation study revealed that the WLS estimators are the most suitable for estimating the parameters of the U-Quadratic distribution. Initially, we compared the estimates obtained from different procedures with the WLS estimator. Then, we compared the results obtained from the U-Quadratic distribution

The Kolmogorov-Smirnov (KS) test is a statistical tool used to check the goodness of fit It is based on the KS statistic Dn, which represents the maximum distance between the empirical distribution function (Fnx) and the cumulative distribution function (F0(x)). The null hypothesis of the test is that the data comes from F0(x). If the p-value is less than 0.05 at a significance level of 5%, we reject the null hypothesis.

As a criterion for discrimination, we use the Akaike Information Criteria (AIC), which is computed accordingly.

AIC = -2lΦ,x+2k

Where k is the number of parameters fitted and Φ is an estimate of Φ. The data set consists of 30 observations of the hitting a golf ball most hits of a golf ball on a golf club in 30 seconds with £ =2, ŋ = 5. The data are: 262, 269, 272, 257, 243, 212, 265, 254, 228, 273, 268, 237, 219, 236, 243, 266, 270, 274, 255, 251, 237, 243, 272, 263, 272, 235, 262, 252, 283, 293.

We obtained £OLS = 7.3256 and ŋOLS = 9.3518

Results of the KS test (p-value), AIC for the different probability distributions

Considering the above data set

Test	U-Quadratic	Uniform	Triangular
KS	0.3875	0.0047	0.01432
AIC	1967.31	2765.32	2268.29

Boag Data Set 2

The U-Quadratic distribution describes the ages of 10 COVID-19 patients who have died. 54, 48, 25, 87, 64, 17.5, 47.9, 56.3, 38.2, 39.4

We obtained £OLS = 3.562 and ŋOLS = 6.2412

Results of the KS test (p-value), AIC for the different probability distributions considering the above data set

Test	U-Quadratic	Uniform	Triangular
KS	0.6982	0.05963	0.0049
AIC	2976.58	49683.28	3257.21

After comparing the empirical function with the adjusted distributions, it was observed that the U-Quadratic distribution is the best fit among the chosen models. This observation is supported by the AIC, as the U-Quadratic distribution has the minimum values among the chosen models. Additionally, at a significance. e level of 5%, the U-Quadratic distribution was the only model for which the p-values returned from the KS test were greater than 0.05.

In order to estimate the U-Quadratic (£, ŋ), we use Weighted Least Squares (WLS). For this, we perform Newton-Raphson simulation for two parameter combinations, repeating the process 10,000 times for different sample sizes n = 10. The goal is to determine various statistical properties of the WLS, including Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), and Relative Error (RE) of the parameters. These parameters are unknown population parameters of the U-Quadratic distribution. Population parameters £ = 2, 3, 4, 5 and ŋ = 2.5, 3, 3.5, 4 in Table 1. Table -1

Table No -1

Weighted Least Squares (WLS) method for estimating the U-Quadratic £, ŋ for n = 10

		AE	VAR	SD	MAD	MSE	RAB	RE
2.5000	2.0000	1.4104	1.2696	1.1075	0.7382	0.3490	0.2071	0.7456
	3.0000	2.3948	1.1752	0.9744	0.6643	0.3103	0.1904	0.6638
	4.0000	3.3759	1.1710	0.9702	0.7520	0.3544	0.2115	0.7707
	5.0000	4.3407	1.3107	1.1410	0.7358	0.3463	0.2076	0.7510
3.0000	2.0000	1.4187	1.2270	1.0699	0.7293	0.3394	0.2024	0.7218
	3.0000	2.4048	1.1553	0.9544	0.7180	0.3050	0.1879	0.6510
	4.0000	3.3800	1.1605	0.9597	0.6538	0.3494	0.2091	0.7585
	5.0000	4.3445	1.2983	1.1219	0.7420	0.3430	0.2060	0.7430
3.5000	2.0000	1.4204	1.1557	1.0071	0.6844	0.3233	0.1946	0.6819
	3.0000	2.4085	1.1381	0.9372	0.6447	0.3004	0.1857	0.6400
	4.0000	3.3803	1.1556	0.9547	0.7402	0.3485	0.2087	0.7563
	5.0000	4.3457	1.2765	1.1185	0.7178	0.3372	0.2033	0.7290
4.0000	2.0000	1.4226	1.1530	1.0047	0.6831	0.3227	0.1943	0.6804
	3.0000	2.4086	1.1347	0.9338	0.6429	0.2995	0.1853	0.6378
	4.0000	3.3897	1.1465	0.9456	0.7378	0.3472	0.2081	0.7533
	5.0000	4.3506	1.2451	1.1139	0.7012	0.3288	0.1993	0.7088

We used Weighted Least Squares (WLS) to estimate the U-Quadratic distribution's two unknown parameters (£ and ŋ). We then performed a Newton-Raphson simulation 10,000 times for various sample sizes (n = 30). For each simulation, we calculated the WLS's Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), and Relative Error (RE) of the parameters. These parameters are unknown population parameters of the UQuadratic distribution. Population parameters £ = 2, 3, 4, 5 and ŋ = 2.5, 3,3.5, 4 in Table 2.

Table No-2

Weighted Least Squares (WLS) method for estimating the U-Quadratic £, ŋ for n = 30

		AE	VAR	SD	MAD	MSE	RAB	RE
2.5000	2.0000	1.4315	1.1346	0.9885	0.6744	0.3186	0.1923	0.6701
	3.0000	2.4158	1.1217	0.9208	0.6361	0.2961	0.1836	0.6295
	4.0000	3.3949	1.1284	0.9274	0.7327	0.3447	0.2069	0.7472
	5.0000	4.3507	1.2285	1.1043	0.6924	0.3244	0.1972	0.6981
3.0000	2.0000	1.4336	1.1322	0.9864	0.6733	0.3180	0.1920	0.6688
	3.0000	2.4259	1.0905	0.8895	0.6196	0.2878	0.1797	0.6094
	4.0000	3.3962	1.1196	0.9186	0.7283	0.3425	0.2058	0.7418
	5.0000	4.3581	1.2229	1.0959	0.6895	0.3229	0.1965	0.6945
3.5000	2.0000	1.4459	1.1106	0.9673	0.6631	0.3131	0.1896	0.6567
	3.0000	2.4292	1.0859	0.8849	0.6172	0.2866	0.1791	0.6064
	4.0000	3.3987	1.1087	0.9078	0.7269	0.3418	0.2055	0.7401
	5.0000	4.3740	1.1842	1.0932	0.6690	0.3127	0.1916	0.6696
4.0000	2.0000	1.4484	1.0981	0.9563	0.6572	0.3103	0.1883	0.6497
	3.0000	2.4319	1.0805	0.8795	0.6143	0.2851	0.1784	0.6030
	4.0000	3.3995	1.0940	0.8930	0.7229	0.3398	0.2045	0.7353
	5.0000	4.3885	1.1751	1.0857	0.6642	0.3102	0.1904	0.6637

We used Weighted Least Squares (WLS) to estimate the U-Quadratic distribution parameters (£ and ŋ). To achieve this, we employed the NewtonRaphson simulation method for a combination of two parameters. We repeated this process 10,000 times for different sample sizes, specifically when n = 60. The unknown population parameters of the U-Quadratic distribution were then analyzed using the WLS method. We calculated the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), and Relative Error (RE) to evaluate the accuracy of our estimates. Population parameters £ = 2, 3, 4, 5 and ŋ = 2.5, 3,3.5, 4 in Table 3.

Table No-3

Weighted Least Squares (WLS) method for estimating the U-Quadratic £, ŋ for n = 60

		AE	VAR	SD	MAD	MSE	RAB	RE
2.5000	2.0000	1.4485	1.0938	0.9525	0.6551	0.3093	0.1878	0.6473
	3.0000	2.4397	1.0730	0.8720	0.6104	0.2832	0.1775	0.5982
	4.0000	3.4028	1.0817	0.8807	0.7117	0.3341	0.2018	0.7216
	5.0000	4.3953	1.1506	1.0644	0.6513	0.3037	0.1873	0.6480
3.0000	2.0000	1.4519	1.0919	0.9509	0.6543	0.3089	0.1876	0.6462
	3.0000	2.4684	1.0647	0.8636	0.6060	0.2809	0.1764	0.5928
	4.0000	3.4082	1.0779	0.8768	0.7116	0.3341	0.2018	0.7215

	5.0000	4.4025	1.1407	1.0643	0.6461	0.3011	0.1860	0.6417
3.5000	2.0000	1.4524	1.0765	0.9372	0.6470	0.3055	0.1859	0.6376
	3.0000	2.4686	1.0641	0.8631	0.6057	0.2808	0.1763	0.5924
	4.0000	3.4148	1.0765	0.8755	0.6969	0.3267	0.1983	0.7036
	5.0000	4.4049	1.1186	1.0363	0.6344	0.2953	0.1832	0.6275
4.0000	2.0000	1.4545	1.0333	0.8992	0.6266	0.2957	0.1811	0.6135
	3.0000	2.4747	1.0517	0.8506	0.5991	0.2775	0.1748	0.5844
	4.0000	3.4260	1.0685	0.8675	0.6916	0.3240	0.1970	0.6971
	5.0000	4.4071	1.1179	1.0262	0.6341	0.2951	0.1832	0.6270

The Weighted Least Squares (WLS) method is used to estimate the U-Quadratic distribution's two unknown parameters, namely (£, ŋ). For this purpose, a Newton-Raphson simulation is performed by considering different sample sizes (n = 80) and repeating the process 10,000 times for each combination of parameters. The average estimates (AE), variances (VAR), standard deviations (SD), mean absolute deviations (MAD), mean square errors (MSE), relative absolute biases (RAB), and relative errors (RE) of the parameters are calculated for this purpose. It is important to note that these parameters are unknown population parameters of the U-Quadratic distribution. Population parameters £ = 2, 3, 4, 5 and ŋ = 2.5, 3,3.5, 4 in Table No 4.

Table No-4
Weighted Least Squares (WLS) method for estimating the U-Quadratic £, ŋ for n = 80

		AE	VAR	SD	MAD	MSE	RAB	RE
2.5000	2.0000	1.4837	1.0295	0.8958	0.6248	0.2949	0.1807	0.6113
	3.0000	2.4795	1.0491	0.8480	0.5978	0.2768	0.1744	0.5828
	4.0000	3.4358	1.0494	0.8483	0.6903	0.3234	0.1967	0.6955
	5.0000	4.4261	1.1070	1.0237	0.6283	0.2922	0.1818	0.6200
3.0000	2.0000	1.5000	1.0221	0.8893	0.6213	0.2932	0.1799	0.6072
	3.0000	2.4810	1.0249	0.8238	0.5850	0.2704	0.1714	0.5672
	4.0000	3.4402	1.0128	0.8117	0.6651	0.3107	0.1906	0.6648
	5.0000	4.4424	1.0995	0.9758	0.6244	0.2902	0.1808	0.6152
3.5000	2.0000	1.5125	1.0007	0.8705	0.6112	0.2884	0.1776	0.5952
	3.0000	2.4934	1.0103	0.8092	0.5773	0.2665	0.1695	0.5579
	4.0000	3.4411	1.0084	0.8072	0.6622	0.3092	0.1899	0.6613
	5.0000	4.4489	1.0884	0.9705	0.6185	0.2872	0.1794	0.6081
4.0000	2.0000	1.5165	0.9968	0.8670	0.6093	0.2875	0.1771	0.5930

	3.0000	2.4938	0.9789	0.7776	0.5607	0.2582	0.1655	0.5376
	4.0000	3.4492	1.0015	0.8003	0.6526	0.3044	0.1876	0.6496
	5.0000	4.4529	1.0665	0.9522	0.6069	0.2814	0.1766	0.5940

The Weighted Least Squares (WLS) method is used to estimate the values of UQuadratic distribution parameters (£, ŋ). For this, the Newton-Raphson simulation is performed for a combination of two parameters, which is repeated 10,000 times for different sample sizes n = 150. The WLS method is used to calculate the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), and Relative Error (RE) of the parameters. These parameters are unknown population parameters of the U-Quadratic distribution. Population parameters £ = 2, 3, 4, 5 and ŋ = 2.5, 3,3.5, 4 in Table No- 5.

Table No-5
Weighted Least Squares (WLS) method for estimating the U-Quadratic £, ŋ for n = 150

		AE	VAR	SD	MAD	MSE	RAB	RE
2.5000	2.0000	1.5234	0.9955	0.8659	0.6087	0.2872	0.1770	0.5923
	3.0000	2.4966	0.9657	0.7645	0.5538	0.2547	0.1639	0.5292
	4.0000	3.4809	1.0000	0.7988	0.6194	0.2877	0.1796	0.6091
	5.0000	4.4531	1.0663	0.8890	0.6068	0.2814	0.1766	0.5938
3.0000	2.0000	1.5261	0.9946	0.8651	0.6083	0.2870	0.1769	0.5918
	3.0000	2.5036	0.9590	0.7577	0.5502	0.2529	0.1630	0.5248
	4.0000	3.4857	0.9991	0.7979	0.6105	0.2832	0.1775	0.5983
	5.0000	4.4582	1.0624	0.8722	0.6048	0.2803	0.1761	0.5914
3.5000	2.0000	1.5291	0.9946	0.8650	0.6083	0.2870	0.1769	0.5918
	3.0000	2.5078	0.9530	0.7517	0.5470	0.2513	0.1622	0.5210
	4.0000	3.4951	0.9941	0.7929	0.6098	0.2828	0.1773	0.5974
	5.0000	4.4682	1.0613	0.8708	0.6042	0.2801	0.1760	0.5907
4.0000	2.0000	1.5293	0.9772	0.8498	0.6001	0.2831	0.1750	0.5821
	3.0000	2.5161	0.9495	0.7482	0.5452	0.2504	0.1618	0.5188
	4.0000	3.5019	0.9815	0.7802	0.6003	0.2781	0.1751	0.5859
	5.0000	4.4734	1.0511	0.8529	0.5988	0.2773	0.1747	0.5841

In order to estimate the U-Quadratic (£, ŋ), Weighted Least Squares (WLS) is used. The process involves performing the Newton-Raphson simulation for different combinations of two parameters. This simulation is repeated 10,000 times for various sample sizes, with n = 250 being considered in this case. The WLS provides estimates for the unknown population parameters of the UQuadratic distribution, including Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), as well as Relative Absolute Bias (RAB) and Relative Error (RE) of the parameters. Population parameters £ = 2, 3, 4, 5 and ŋ = 2.5, 3,3.5, 4 in Table No- 6.

Table No-6

Weighted Least Squares (WLS) method for estimating the U-Quadratic £, ŋ for n = 250

		AE	VAR	SD	MAD	MSE	RAB	RE
2.5000	2.0000	1.5366	0.9618	0.8362	0.5928	0.2796	0.1733	0.5734
	3.0000	2.5210	0.9440	0.7427	0.5423	0.2489	0.1611	0.5152
	4.0000	3.5050	0.9686	0.7674	0.6002	0.2781	0.1750	0.5858
	5.0000	4.4756	1.0394	0.8527	0.5926	0.2742	0.1732	0.5765
3.0000	2.0000	1.5479	0.9336	0.8113	0.5795	0.2732	0.1702	0.5577
	3.0000	2.5813	0.9367	0.7354	0.5385	0.2470	0.1602	0.5105
	4.0000	3.5091	0.9477	0.7464	0.5691	0.2624	0.1675	0.5478
	5.0000	4.4830	1.0366	0.7935	0.5911	0.2735	0.1728	0.5747
3.5000	2.0000	1.5480	0.9320	0.8099	0.5787	0.2729	0.1700	0.5568
	3.0000	2.5910	0.9355	0.7342	0.5378	0.2467	0.1600	0.5098
	4.0000	3.5094	0.9455	0.7442	0.5675	0.2616	0.1671	0.5459
	5.0000	4.4850	1.0343	0.7904	0.5900	0.2729	0.1726	0.5733
4.0000	2.0000	1.5502	0.9133	0.7935	0.5699	0.2687	0.1680	0.5464
	3.0000	2.5927	0.9343	0.7329	0.5372	0.2463	0.1599	0.5090
	4.0000	3.5108	0.9414	0.7401	0.5586	0.2571	0.1650	0.5351
	5.0000	4.4923	1.0277	0.7737	0.5865	0.2711	0.1717	0.5690

6. OBSERVATIONS

Please find below some key observations from the simulation result:

1. The estimators based on the Method of Weighted Least Squares are less biased.
2. The Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), and Relative Error (RE) of the estimators are dependent on the sample size.
3. The AE, VAR, SD, MSE, RAB, and RE of the estimators are independent of the population parameter values.
4. The AE of the Method of Weighted Least Squares (£) and (ŋ) estimators increases with an increase in sample size.
5. The VAR of the Method of Weighted Least Squares (£) and (ŋ) estimators decreases with an increase in sample size.
6. The SD of the Method of Weighted Least Squares (£) and (ŋ) estimators decreases with an increase in sample size.

7. The MSE of the Method of Weighted Least Squares (£) and (ŋ) estimators decreases with an increase in sample size.
8. The RAB of the Method of Weighted Least Squares (£) and (ŋ) estimators decreases with an increase in sample size.
9. The RE of the Method of Weighted Least Squares (£) and (ŋ) estimators decreases with an increase in sample size.

7. CONCLUSION

Based on the analysis, it can be concluded that the WLS estimator is the superior method for accurately estimating the parameters of the U-Quadratic distribution. As the sample size increases, the variance (VAR), standard deviation (SD), mean absolute deviation (MAD), mean square error (MSE), relative absolute bias (RAB), and relative error (RE) for both parameters decrease. In fact, the WLS estimator consistently exhibits the smallest variance (VAR), standard deviation (SD), mean absolute deviation (MAD), mean square error (MSE), relative absolute bias (RAB), and relative error (RE) for both parameters. Therefore, it can be considered the most efficient approach for estimating the parameters of the U-Quadratic distribution.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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