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Reliability and Availability Analysis of a Triplex Sensor Node System with Shared Repair

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ABSTRACT

Wireless sensor nodes are prone to failures due to severe resource constraints and usually harsh operational environments. Many wireless sensor network applications are mission-critical, requiring continuous operation. Thus, in order to meet application requirements reliably, it is imperative to design fault-tolerance into wireless sensor networks (WSNs). In this study, we deal with the reliability and availability analysis of a triplex repairable wireless sensor node system under a shared repair facility. The repair facility is turned on when a sensor node fails, providing repair under a first-fail-first-repair policy. We analyze system mean time to failure (MTTF) and steady-state availability (SSA) as a function of the component failure and repair rates. Our primary objective is to provide explicit expressions for these performance measures and highlight the significance of fault-tolerance into WSNs.

KEYWORDS: *wireless sensor network, fault-tolerance, reliability, MTTF, SSA.*

I. INTRODUCTION

Technological advancements in micro – electro - mechanical systems (MEMS), wireless communications and digital electronics have led to the proliferation of Wireless Sensor Networks (WSNs) in a wide variety of application domains such as healthcare, environmental monitoring, home security and mission-critical military operations. Thus, offering a remarkable potential to bridge the gap between the physical world of sensors and a virtual world of information and services. Due to severe resource constraints, and usually unattended and harsh operational environments, sensor nodes are prone to failures. Manual inspection of faulty sensor nodes after deployment is typically impractical. Nevertheless, since many WSN applications are mission-critical requiring continuous operation, it imperative to design fault-tolerance into WSNs so that the overall sensor network functionalities can be sustained without interruption despite sensor node failures [1-4].

Considerable research efforts have been expended in designing fault-tolerance into WSNs. In [1], a set of fault-tolerant models using cold spares are proposed and their reliability performance compared with that of models using hot spares. In addition, two types of back-up strategies (*homogeneous* and *heterogeneous*) were investigated. Silva *et al* [5] proposed a methodology based on an automatic generation of a fault tree to evaluate the reliability and availability of wireless sensor networks when permanent faults occur in network devices. In [6], a reliable and fault-tolerant Markov model for a sensor network system using different types of sensors and spares that replace sensors in case failure occurs was presented. In [7], Bein *etal* emphasized the importance of heterogeneous fault-tolerance by exploring reliability issues in multi-fusion sensor networks. Munir and Gordon-Ross [2] investigated the synergy of fault detection and fault-tolerance for WSNs and proposed a fault-tolerant sensor node model consisting of duplex sensors which exploits this synergy between fault detection and fault-tolerance. In [8], Markov chains are used in evaluating the reliability of sensor networks. Though inspired by the work in [2], their model removes the constraint of having mutually exclusive clusters (i.e., a node can be counted in two clusters) allowing for a more flexible reliability analysis.

In this paper, a fault-tolerant model of a triplex repairable wireless sensor node system is presented, and reliability and availability performance of the model evaluated. The rest of this paper is organized as follows: Section II describes the modeling assumptions. Using Markov process theory, Laplace transforms and limiting state probability evaluation, the explicit expressions of the mean time to failure (MTTF) and the steady-state availability (SSA) for the system are obtained in Sections III and IV, respectively. In Section V, the influence of system parameters on the MTTF and SSA is studied. Conclusions are drawn in Section VI.

II. Modeling Assumptions

The following high-level assumptions of a triplex sensor node system, consisting of three active sensor nodes connected in parallel and a single repair facility were made when building the model:

i. The sensor nodes are independent and identical, each with a failure rate, λ and the rate at which a faulty sensor node returns from any given failed state to the preceding working state is μ , the repair rate.

ii. Each sensor node can exist in one of two states (operating and failed).

iii. When an active sensor node fails, it is immediately repaired if the repair facility is idle. Repair is performed based on a first-fail-first-repair policy.

iv. A fault detection algorithm continously monitors the sensor nodes to identify failures.

v. Environmental and operational conditions for the system are relatively stable as a function of time.

III. Reliability and mean time to failure

The state transition diagram in figure 1 shows the Markov model for our proposed reliability model.

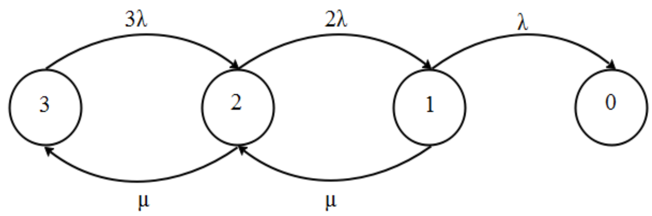


Figure 1: Triplex Sensor Node Reliability Model with Shared Repair

The states in the Markov model represent the number of operating or good sensor nodes. The differential equations describing the sensor node triplex Markov model are:

$$P_0'(t) = \lambda P_1(t)$$
$$P_1'(t) = 2\lambda P_2(t) - (\lambda+\mu)P_1(t)$$
$$P_2'(t) = 3\lambda P_3(t) - (2\lambda+\mu)P_2(t) + \mu P_1(t)$$
$$P_3'(t) = -3\lambda P_3(t) + \mu P_2(t) \qquad (1)$$

where $P_i(t)$ denotes the probability that the sensor node will be in state i at time t, and $P_i'(t)$ represents the first order derivative of $P_i(t)$. When all 3 sensor nodes have failed (i.e., the absorbing state), the system is considered to have failed and no recovery is possible.

Equation (1) may be expressed in matrix form as:

$$\begin{bmatrix} P_0'(t) & P_1'(t) & P_2'(t) & P_3'(t) \end{bmatrix} = \begin{bmatrix} P_0(t) & P_1(t) & P_2(t) & P_3(t) \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda & -(\lambda+\mu) & \mu & 0 \\ 0 & 2\lambda & -(2\lambda+\mu) & \mu \\ 0 & 0 & 3\lambda & -3\lambda \end{bmatrix}$$

Since the intensity matrix,

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda & -(\lambda+\mu) & \mu & 0 \\ 0 & 2\lambda & -(2\lambda+\mu) & \mu \\ 0 & 0 & 3\lambda & -3\lambda \end{bmatrix}$$

does not have full rank [9], the matrix equations can be reduced by removing the row and column corresponding to the absorbing state without losing any information about $P_0(t), P_1(t), P_2(t)$ and $P_3(t)$.

If we assume that the system is initially working, then

$$P_i(t=0)=\begin{cases}1, \text{ if } i=3\\ 0, \text{ otherwise}\end{cases} \quad (2)$$

Hence, the system differential equations employing Laplace transform are:

$$-(\lambda+\mu)P_1^*(s)+2\lambda P_2^*(s)=sP_1^*(s)$$
$$\mu P_1^*(s)-(2\lambda+\mu)P_2^*(s)+3\lambda P_3^*(s)=sP_2^*(s)$$
$$\mu P_2^*(s)-3\lambda P_3^*(s)=sP_3^*(s)-1 \quad (3)$$

Solving these equations leads to $P_i^*(s)$

For a system that is in a specified operating or functioning state at time $t=0$, the relaibility $R(t)$ determines the probability that a system does not leave the set B of functioing states during the time interval $[0, \Delta t]$. Thus the reliability is

$$R(t)=\sum_{i\in B}P_i(t) \quad (4)$$

The Laplace transform of the reliability is expressed as:

$$R^*(s)=\sum_{i\in B}P_i^*(s) \quad (5)$$

The MTTF is determined by

$$MTTF=\int_0^{\infty}R(t)dt=R^*(s)\big|_{s=0} \quad (6)$$

Therefore,

$$MTTF_{Shared\ Repair}=\frac{11\lambda^2}{6\lambda^3}+\frac{4\lambda\mu+\mu^2}{6\lambda^3}$$

In the absence of a repair facility $(\mu=0)$,

$$MTTF_{Non-repairable}=\frac{11\lambda^2}{6\lambda^3}$$

IV. Availability and Steady-state availability

For a system that can tolerate brief failures and continue its operations after it is repaired, availability is a useful measure of its performance [10]. Figure 2 shows the behaviour of the system which is initially good.

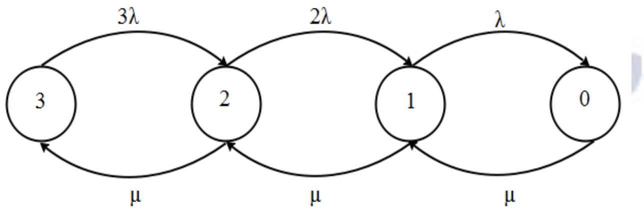


Figure 2: Triplex Sensor Node Availability Model with Shared Repair

The probabilities for either remaining or moving to all the various states during the interval $[0, \Delta t]$ is described by the stochastic transitional probability matrix, $P_{ij}\Delta t$. Hence,

$$\left[P_{ij}\Delta t\right]=\begin{bmatrix}1-3\lambda\Delta t & 3\lambda\Delta t & 0 & 0\\ \mu\Delta t & 1-(2\lambda+\mu)\Delta t & 2\lambda\Delta t & 0\\ 0 & \mu\Delta t & 1-(\lambda+\mu)\Delta t & \lambda\Delta t\\ 0 & 0 & \mu\Delta t & 1-\mu\Delta t\end{bmatrix} \quad (7)$$

for $i, j=0,1,2,3$.

If α is defined as the limiting state probability vector which remains unchanged when multiplied by $P_{ij}\Delta t$ then [11],

$$\alpha P_{ij}\Delta t=\alpha \quad (8)$$

If α is given by $\left[P_3(t) \quad P_2(t) \quad P_1(t) \quad P_0(t)\right]$ for the triplex repairable system, then

$$P_1(t)\lambda\Delta t+P_0(t)(1-\mu\Delta t)=P_0(t)$$
$$P_2(t)2\lambda\Delta t+P_1(t)\left[1-(\lambda+\mu)\Delta t\right]+P_0(t)\mu\Delta t=P_1(t)$$
$$P_3(t)3\lambda\Delta t+P_2(t)\left[1-(2\lambda+\mu)\Delta t\right]+P_1(t)\mu\Delta t=P_2(t)$$
$$P_3(t)(1-3\lambda\Delta t)+P_2(t)\mu\Delta t=P_3(t) \quad (9)$$

The steady-state availability of the system is the limiting value as $\Delta t\to\infty$ of the probability that the system will function at time Δt. Thus,

$$\lim_{\Delta t\to\infty}\begin{bmatrix}dP_0(t)/dt\\ dP_1(t)/dt\\ dP_2(t)/dt\\ dP_3(t)/dt\end{bmatrix}=\lim_{\Delta t\to\infty}\begin{bmatrix}-\mu & \lambda & 0 & 0\\ \mu & -(\lambda+\mu) & 2\lambda & 0\\ 0 & \mu & -(2\lambda+\mu) & 3\lambda\\ 0 & 0 & \mu & -3\lambda\end{bmatrix}\begin{bmatrix}P_0(t)\\ P_1(t)\\ P_2(t)\\ P_3(t)\end{bmatrix}$$

As $\Delta t\to\infty$, $P_i(t)\to$ constant and $dP_i(t)/dt\to 0$ which leads to an unsolvable system. To avoid this difficulty, an additional equation is introduced while deleting one of the original equations and a new system formed:

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\ \mu & -(\lambda+\mu) & 2\lambda & 0\\ 0 & \mu & -(2\lambda+\mu) & 3\lambda\\ 0 & 0 & \mu & -3\lambda\end{bmatrix}\begin{bmatrix}P_0(t)\\ P_1(t)\\ P_2(t)\\ P_3(t)\end{bmatrix} \quad (10)$$

Hence, the SSA of the triplex wireless sensor node system is given by

$$SSA_{Shared\ Repair}=P_1(t)+P_2(t)+P_3(t)$$
$$=\frac{6\lambda^2\mu+3\lambda\mu^2+\mu^3}{6\lambda^3+6\lambda^2\mu+3\lambda\mu^2+\mu^3}$$

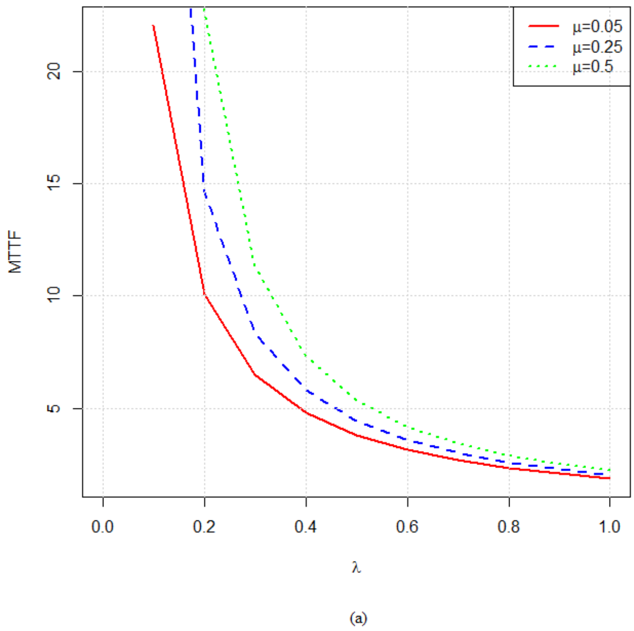
and the unavailability,

$$U(t)=P_0(t)$$
$$=\frac{6\lambda^3}{6\lambda^3+6\lambda^2\mu+3\lambda\mu^2+\mu^3}$$

V. Results and Discussion

We used the Maple and R Software Packages to obtain our model results. We compared the MTTF and SSA for different values of λ and μ.

In Figure 3(a) and (b), the MTTF is plotted with respect λ and μ respectively. As expected, in 3(a), the behaviour of the MTTF with λ is descending. For a given value of μ, the MTTF decreases as λ increases. Figure 3(b) indicates that the behaviour of the MTTF is ascending with respect to μ. When $\mu = 0$, the system is non-repairable. In which case MTTF = 26.19, 30.56 and 36.67 for $\lambda = 0.07$, $\lambda = 0.06$ and $\lambda = 0.05$ respectively. With increasing value of μ the system becomes reliable. However, for a fixed value of μ, the MTTF declines as λ increases.



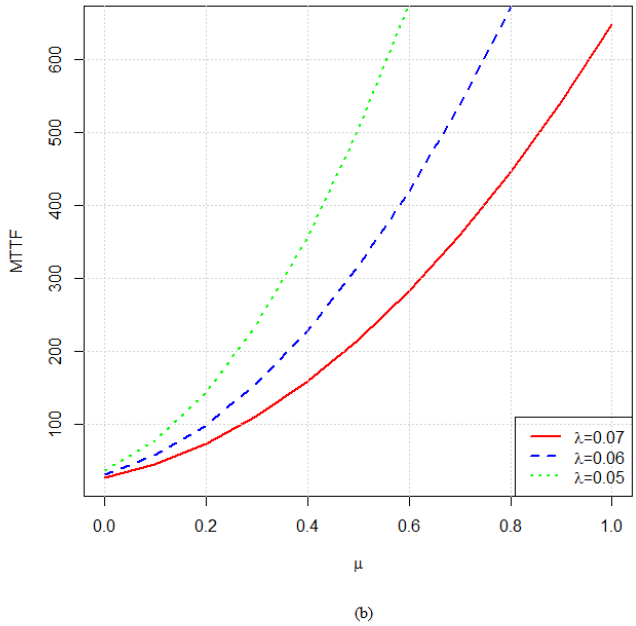
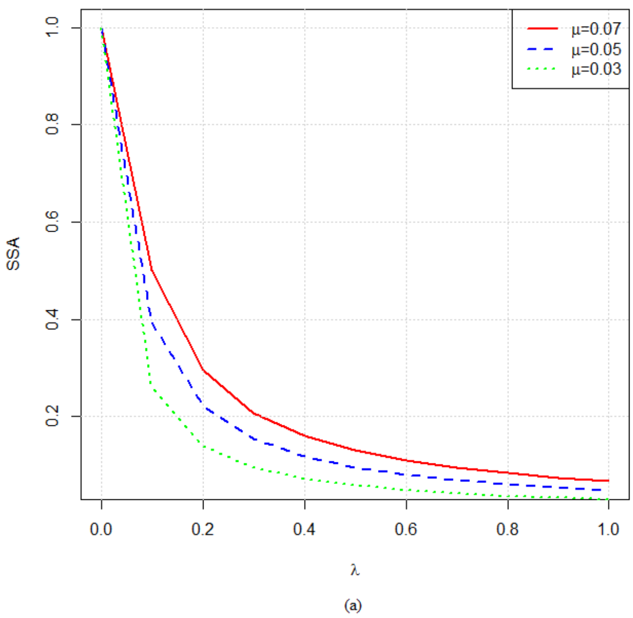


Figure 3: Variations of MTTF with parameters (a) λ and (b) μ

The steady-state availability versus λ and μ is graphically illustrated in Figure 4(a) and (b) respectively. As shown, the behaviour of the SSA is descending with λ and ascending with μ. In the case of $\lambda = 0$ which is equivalent to the case of considerably increasing μ, the system becomes reliable and the SSA equals 1. In 4(a), considering a constant value of λ, the SSA increases with increasing μ. In 4(b), for a fixed value of μ, the SSA declines as λ gets larger.



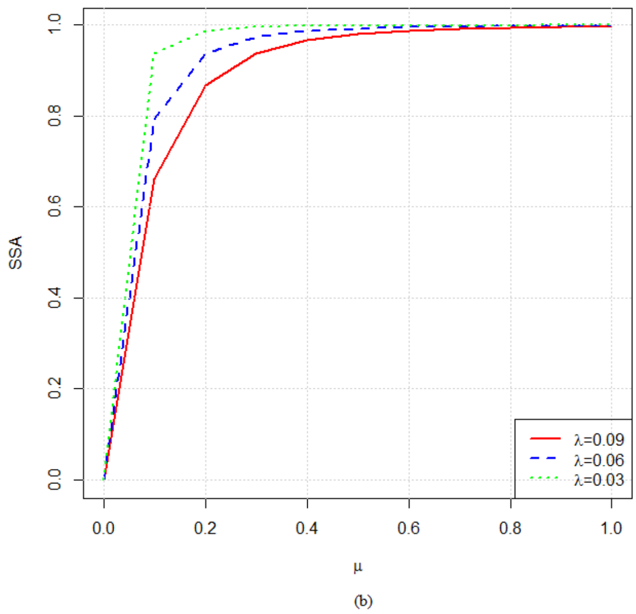


Figure 4: Variations of steady-state availability with parameters (a) λ and (b) μ

VI. Conclusion

In this study we investigated the MTTF and SSA for a triplex repairable wireless sensor node system consisting of three identical sensor nodes and a single repair facility. It was assumed that in case an active sensor node fails, it can be immediately repaired if the repair facility is idle. Using Markov process theory, Laplace transforms and limiting state probability evaluation, the explicit expressions of the MTTF and SSA were derived analytically. Analysis of the MTTF and SSA characteristics of the system with respect to λ and μ were performed. It was illustrated that the MTTF and SSA indexes are very sensitive to changes in λ and μ, and also, that the effect of the repair facility increased the mean life by $\frac{4\lambda\mu+\mu^2}{6\lambda^3}$.

The MTTF and SSA expressions and comparison results generated from this study serve as a contributing effort to explore fault-tolerance issues in wireless sensor networks.

References

[1] L. Xing, H. Li, and H. E. Michel, "Fault-Tolerance and Reliability Analysis for Wireless Sensor Networks," *International Journal of Performance Engineering*, vol. 5, no. 5, pp. 419–431, 2009.

[2] A. Munir and A. Gordon-Ross, "Markov Modeling of Fault-Tolerant Wireless Sensor Networks," in *20th International Conference on Computer Communications and Networks (ICCCN)*, 2011, vol. 3, pp. 1–6.

[3] I. Koren and C. M. Krishna, *Fault Tolerant Systems*. Morgan Kaufmann, 2007.

[4] M. Katiyar, H. P. Sinha, and D. Gupta, "On Reliability Modeling in Wireless Sensor Networks-A Review," *International Journal of Computer Science Issues*, vol. 9, no. 6, pp. 99–105, 2012.

[5] I. Silva, L. A. Guedes, P. Portugal, and F. Vasques, "Reliability and Availability Evaluation of Wireless Sensor Networks for Industrial Applications," *Sensors*, vol. 12, no. 1, pp. 806–838, 2012.

[6] V. Kumar, R. B. Patel, M. Singh, and R. Vaid, "Reliability Analysis in Wireless Sensor Network," *International Journal of Engineering and Technology*, vol. 3, no. 2, pp. 74–79, 2011.

[7] D. Bein, V. Jolly, B. Kumar, and S. Latifi, "Reliability Modeling in Wireless Sensor Networks," *International Journal of Information Technology*, vol. 11, no. 2, pp. 1–9, 2005.

[8] T. Arjannikov, S. Diemert, S. Ganti, C. Lampman, and E. C. Wiebe, "Using Markov Chains to Model Sensor Network Reliability," in *12th International Conference on Availability, Reliability and Security*, 2017.

[9] M. Rausand and A. Hoyland, *System Reliability Theory: Models, Statistical Methods, and Applications*, 2nd Ed. Hoboken, New Jersey: John Wiley & Sons, Inc., 2004.

[10] M. Amiri, V. Přenosil, "General Solutions for MTTF and Steady-State Availability of NMR Systems," in *9th International Symposium on Reconfigurable and Communication-Centric Systems-on-Chip (ReCoSoC)*, 2014.

[11] R. Billinton and R. N. Allan, *Reliability Evaluation of Engineering Systems: Concepts and Techniques*, 2nd Ed. New York: Springer Science+Business Media, LLC, 1992.