Available online at: http://www.ijmtst.com/vol5issue06.html
International Journal for Modern Trends in Science and Technology
ISSN: 2455-3778 :: Volume: 05, Issue No: 06, June 2019

Torsional Wave Propagation in a Porothermoelastic Hollow Cylinder

Manjula Ramagiri

Department of Mathematics, Kakatiya University, Telangana State, India.

To Cite this Article
Manjula Ramagiri, "Torsional Wave Propagation in a Porothermoelastic Hollow Cylinder", *International Journal for Modern Trends in Science and Technology*, Vol. 05, Issue 06, June 2019, pp.-27-30.

Article Info
Received on 10-May-2019, Revised on 29-May-2019, Accepted on 08-June-2019.

ABSTRACT

This paper presents a theoretical study of torsional vibrations of porothermoelastic hollow cylinder. The governing equations are solved to obtain the general solution. The frequency equation is obtained when boundaries are free and mixed boundary conditions.

KEYWORDS: *Porothermoelasticity, Cylinder, torsional vibrations, Frequency equation.*

I. Introduction

Wave propagation in porothermoelasticity have many applications such as Seismology, Engineering. Torsional vibrations of shafts are important in Engineering in particular in ship engines and aeroengines. Employing Biot's theory [1] torsional vibrations of finite hollow poroelastic circular cylinder is studied by Tajuddin and Sarma [2]. Torsional vibrations of poroelastic cylinder is investigated by Tajuddin and Sarma [3]. Tajuddin and Shah [4] studied torsional vibrations of infinite hollow poroelastic cylinders. Torsional wave propagation in a poroelastic hollow circular cylinder is analysed by Irabhim Abbas [5]. Study of torsional vibrations in an initial stressed composite poroelastic cylinders is studied by Sandhya Rani et. al [6]. Previous studies of torsional vibrations have focused on hollow cylinder. Singru et. al [7] investigated on thermal stress analysis of thick hollow cylinder. Mathematical modeling of thermoelastic state of thick hollow cylinder with nonhomogeneous material properties is studied by Manthena et.al [8]. Thermoelastic interaction in an infinite long hollow cylinder with fractional heat conduction equation is presented by Ahmed E, Abouelregal [9]. Jadhav et. al [10] studied an inverse thermoelastic problem of finite length thick hollow cylinder with internal heat sources. In the above papers thermoelasticity is considered for hollow cylinders. Porthermoelastic analysis of anisotropic hollow cylinders with applications is studied by Mazen Kanj et. al [11]. Bing Bai [12] investigated the consolidation solutions of a saturated porothermoelastic hollow cylinder with infinite length. Jabbari and Dehbani [13] studied an exact solution for classic coupled thermoporoelasticity in axisymmetric cylinder. Asphaltic material in the context of generalized porothermoelasticity is investigated by Mohammed H. Alawi [14]. In the above cited papers torsional vibrations are not considered. In the present paper torsional vibrations is studied for porothermoelastic hollow cylinder.

The rest of the paper is organized as follows. In section 2, governing equations and solution of the problem are given. In section 3, boundary conditions and frequency equation and cases are presented. Finally, conclusions are given in section 4.

II. GOVERNING EQUATIONS AND SOLUTION OF THE PROBLEM

Let (r,θ,z) be the cylindrical polar coordinates. Consider an isotropic hollow cylinder with inner and outer radii a and b respectively. The dynamic equations in polar coordinate system in the absence of body forces [2] and heat conduction [15] for one temperature are as follows.

$$N\nabla^2\vec{u}+\nabla[(A+N)e+Q\varepsilon] = \frac{\partial^2}{\partial t^2}\left(\rho_{11}\vec{u}+\rho_{12}\vec{U}\right)+b\frac{\partial}{\partial t}\left(\vec{u}-\vec{U}\right)$$

$$\nabla[Qe+R\varepsilon]=\frac{\partial^2}{\partial t^2}\left(\rho_{12}\vec{u}+\rho_{22}\vec{U}\right)-b\frac{\partial}{\partial t}\left(\vec{u}-\vec{U}\right).$$

$$K\nabla^2 T=\rho c_v(T+\tau_0\frac{\partial T}{\partial t})\frac{\partial}{\partial t} +\beta T_0(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\nabla.u \qquad (1)$$

Where ∇^2 is the Laplace operator, $\vec{u}=(u,v,w)$ and $\vec{U}=(U,V,W)$ are solid and fluid displacements, e and ε are the dilatations of solid and fluid respectively; the symbols A,N,Q,R are all poroelastic constants; ρ_{ij} are mass coefficients. ρ is the mass density, c_v is the specific heat capacity and K is the thermal conductivity, τ_0 is the relaxation time, The constitutive relations are

$$\sigma_{ij}=2Ne_{ij}+(Ae+Q\varepsilon)\delta_{ij}-\beta T\delta_{ij},(i,j=r,\theta,z),$$
$$s=Qe+R\varepsilon. \qquad (2)$$

In eq. (4), e_{ij}'s strain components, β is the thermal stress, T is the temperature, a δ_{ij} is thewell-known Kronecker delta function. In the above e_{ij}'s are strain components given by

$$e_{r\theta}=\frac{1}{2}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right),\qquad e_{\theta z}=\frac{\partial v}{\partial z} \qquad (3)$$

In the case of torsional vibrations, equations of motion are reduced to the following equation:

$$\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{\partial\sigma_{\theta z}}{\partial z}+\frac{2\sigma_{r\theta}}{r}=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{12}V),$$
$$0=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V),$$
$$K\nabla^2 T=\rho c_v(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}) \qquad (4)$$

For harmonic torsional vibrations, the equations (4) may satisfy by taking

$$v(r)=v(r)e^{i(kz-\omega t)},$$
$$V(r)=V(r)e^{i(kz-\omega t)},$$
$$T(r)=T(r)e^{i(kz-\omega t)}. \qquad (5)$$

In the above k is the wavenumber, ω is the frequency and t is the time Substituting eq. (5) into eq. (4) we obtain.

$$\frac{d^2v}{dr^2}+\frac{1}{r}\frac{dv}{dr}+(q^2-\frac{1}{r^2})v=0,$$
$$V=-\frac{\rho_{12}}{\rho_{22}}$$
$$\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}+(m^2)T=0. \qquad (6)$$

Where

$$q^2=\frac{\omega^2}{v_s^2}-k^2,\quad m^2=\frac{\rho c_v\tau_0}{K}-k^2.$$

The solutions of eq. (6) are

$$v(r)=AJ_1(qr)+BY_1(qr),$$
$$T(r)=AJ_1(mr)+BY_1(mr),$$
$$V(r)=-\frac{\rho_{12}}{\rho_{22}}(AJ_1(qr)+BY_1(qr)). \qquad (7)$$

The non zero stresses are

$$\sigma_{r\theta}=N[Aq\{\frac{1}{rq}J_1(qr)-J_2(qr)-\frac{1}{rq}J_1(qr)\} +Bq\{\frac{1}{rq}Y_1(qr)-Y_2(qr)-\frac{1}{rq}Y_1(qr)\}]e^{i(kz-\omega t)},$$
$$T_r=m[A\frac{1}{mr}J_1(mr)-J_2(mr) +B\frac{1}{mr}Y_1(mr)-Y_2(mr)]e^{i(kz-\omega t)}. \qquad (8)$$

III. BOUNDARY CONDITIONS AND FREQUENCY EQUATION

Free surface traction:

The boundaries $r = a$ and $r = b$ of the cylinder which separates the body from the free space are stress free, then the required boundary condition are

$$\sigma_{r\theta} - T_r = 0 \text{ at } r = a \text{ and } r = b \tag{9}$$

A non trivial solution can be obtained if the determinant of the coefficient matrix vanishes. Accordingly, we obtain the following frequency equation:

$$\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} = 0, \tag{10}$$

where

$$A_{11} = Nq\{\frac{1}{aq}J_1(qa) - J_2(qa) - \frac{1}{rq}J_1(qa)\}$$
$$-m\{\frac{1}{mr}J_1(ma) - J_2(ma)\},$$
$$A_{12} = Nq\{\frac{1}{aq}Y_1(qa) - Y_2(qa) - \frac{1}{rq}Y_1(qa)\}$$
$$-m\{\frac{1}{mr}Y_1(ma) - Y_2(ma)\},$$

A_{21}, A_{22} are similar expression as A_{11}, A_{12} with a is replaced by b.

Fixed surface

The frequency equation for the boundary conditions which specify that the fixed inner and outer surface of the hollow cylinder are fixed displacements

$$v(r) - T(r) = 0 \text{ at } r = a \text{ and } v(r) - T(r) = 0 \text{ at } r = b \tag{11}$$

From eq. (7) we obtain two homogeneous equations

$$A[J_1(qa) - J_1(ma)] + B[Y_1(qa) - Y_1(ma)] = 0,$$
$$A[J_1(qb) - J_1(mb)] + B[Y_1(qb) - Y_1(mb)] = 0.$$

Eliminating the constant A and B, we obtain the frequency equation as

$$\begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix} = 0, \tag{12}$$

Where

$$B_{11} = J_1(qa) - J_1(ma), \quad B_{12} = Y_1(qa) - Y_1(ma),$$
$$B_{21} = J_1(qb) - J_1(mb), \quad B_{22} = Y_1(qb) - Y_1(mb).$$

Inner surface fixed and outer surface free:

The frequency equation for the boundary conditions which specify that the inner surface fixed and outer surface free of the hollow porothermoelastic cylinder

$$v(r) - T(r) = 0 \text{ at } r = a \text{ and } \sigma_{r\theta} - T_r = 0 \text{ at } r = b \tag{13}$$

From eq. (7) and eq. (8) we obtain two homogeneous equations

$$A[J_1(qa) - J_1(ma)] + B[Y_1(qa) - Y_1(ma)] = 0,$$
$$A\{N[q\{\frac{1}{bq}J_1(qb) - J_2(qb) - \frac{1}{bq}J_1(qb)\}$$
$$-m[\frac{1}{mb}J_1(mb) - J_2(mb)]]\} + B[q\{\frac{1}{bq}Y_1(qb)$$
$$-Y_2(qb) - \frac{1}{bq}Y_1(qb)\} - \frac{1}{mb}Y_1(mb)$$
$$-Y_2(mb)] = 0.$$

Eliminating the constant A and B, we obtain the frequency equation a

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = 0, \tag{14}$$

Where

$$C_{11} = J_1(qa) - J_1(ma), \quad C_{12} = Y_1(qa) - Y_1(ma),$$
$$C_{21} = \{N[q\{\frac{1}{bq}J_1(qb) - J_2(qb) - \frac{1}{bq}J_1(qb)\}$$
$$-m[\frac{1}{mb}J_1(mb) - J_2(mb)],$$
$$C_{22} = q\{\frac{1}{bq}Y_1(qb) - Y_2(qb) - \frac{1}{bq}Y_1(qb)\}$$
$$-\frac{1}{mb}Y_1(mb) - Y_2(mb)].$$

Inner surface free and outer surface fixed:

The frequency equation for the boundary conditions which specify that the inner surface free and outer surface fixed of the hollow porothermoelastic cylinder

$$\sigma_{r\theta} - T_r = 0 \text{ at } r = a \text{ and } v(r) - T(r) = 0 \text{ at } r = b \tag{15}$$

From eq. (7) and eq. (8) we obtain two homogeneous equations

$$A\{N[q\{\frac{1}{aq}J_1(qa)-J_2(qa)-\frac{1}{aq}J_1(qa)\}$$
$$-m[\frac{1}{ma}J_1(ma)-J_2(ma)]]\}+B[q\{\frac{1}{aq}Y_1(qa)$$
$$-Y_2(qa)-\frac{1}{aq}Y_1(qa)\}-\frac{1}{ma}Y_1(ma)-Y_2(ma)]=0.$$
$$,A[J_1(qb)-J_1(mb)]+B[Y_1(qb)-Y_1(mb)]=0.$$

Eliminating the constant A and B, we obtain the frequency equation as

$$\begin{vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{vmatrix} = 0, \qquad (16)$$

Where

$$D_{11}=\{N[q\{\frac{1}{aq}J_1(qa)-J_2(qa)-\frac{1}{aq}J_1(qa)\}$$
$$-m[\frac{1}{ma}J_1(ma)-J_2(ma)],$$
$$D_{12}=q\{\frac{1}{aq}Y_1(qa)-Y_2(qa)-\frac{1}{aq}Y_1(qa)\}$$
$$-\frac{1}{ma}Y_1(ma)-Y_2(ma),$$
$$D_{21}=J_1(qb)-J_1(mb), \quad D_{22}=Y_1(qb)-Y_1(mb).$$

IV. Conclusion

Torsional vibrations of porothermoelastic hollow cylinder are investigated. The frequency equation is obtained when the boundaries are free surface, fixed surface, inner surface fixed and outer surface is free, outer surface is fixed and inner surface is free. All the results were presented theoretical

.

References

[1] Biot MA (1956). The theory of propagation of elastic wave in fluid-saturated porous solid. Journal of the Acoustical Society of America. 28: 168-178.

[2] Tajuddin , Sarma (1978). Torsional vibrations of finite hollow poroelastic circular cylinder, Defence Science Journal. 28(2): 97-102.

[3] Tajuddin, Sarma (1980). Torsional vibrations of poroelastic cylinder, Journal of Applied Mechanics. 47:214-216.

[4] Tajuddin , Shah (2007). Torsional vibrations of infinite hollow poroelastic cylinders, Journal of Mechanics of Materials and Structures. 2(1) :189-200.

[5] Ibrahim Abbas (2012). Torsional wave propagation in a poroelastic hollow circular cylinder, Journal of Physics. 1(3): 4-8.

[6] Sandhya Rani, Balu, MallaReddy (2015). Study of torsional vibrations in an initial stressed composite poroelastic cylinders, Journal of Physics: Conference series, 662(2015)012006,doi:10.1088/1742-6596/662/1/012006.

[7] Singru, Sachin, Khobragade (2015). Thermal stress analysis of a thick hollow cylinder, International Journal of Engineering and Innovation Technology. 5(3): 157-160.

[8] Manthena, Lamba Kedar (2018). Mathematical modeling of thermoelastic state of thick hollow cylinder with nonhomogeneous material properties. Journal of Solid Mechanics, 10(1):142-156.

[9] Ahmed E, Abouelregal, (2017). Thermoelastic interaction in an infinite long hollow cylinder with fractional heat conduction equation, Advances in Applied Mathematics and Mechanics. 9(3):378-392.

[10] Jadhav, Ahirrao, Khobragade (2013). An inverse thermoelastic problem of finite length thick hollow cylinder with internal heat sources, Advances in Applied Science Research. 4(3): 302-314.

[11] Mazen Kanj, Youane, Abouseleiman (2005). Porthermoelastic analysis of anisotropic hollow cylinders with applications, International Journal for Numerical and Analytical methods in Geomechanics. 29(2): 103-126.

[12] Bing Bai (2010). Consolidation solutions of a saturated porothermoelastic hollow cylinder with infinite length, International Journal of Engineering Science. 2(1): 37-45, doi: 10.4236/eng2010.21005.

[13] Jabbari and Dehbani (2010). An exact solution for classic coupled thermoporoelasticity in axisymmetric cylinder, Journal of Solid Mechanics. 2(2) :129-143.

[14] Mohammed H. Alawi (2017). Asphaltic material in the context of generalized porothermoelasticity, International Journal on Soft Computing. 8(1): 27-43.

[15] Sachin Kaushal, Rajneesh Kumar and AseemMiglani. (2011).Wave propagation in temperature rate dependent thermoelasticity with two temperatures. Mathematical Sciences, 5(2):125-146.