Comparative Analysis of PID, SMC, SMC with PID Controller for Speed Control of DC Motor

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ABSTRACT

In this thesis, sliding mode control (SMC) technique is used to control the speed of DC motor. The performance of the SMC is judged via MATLAB simulations using linear model of the DC motor and known disturbance. SMC is then compared with PID controller. The simulation result shows that the sliding mode controller (SMC) is superior controller than PID for the speed control of DC motor. Since the SMC is robust in presence of disturbances, the desired speed is perfectly tracked. The sliding mode control (SMC) can adapt itself to the parameter variations and external disturbances, problem of chattering parameter, resulting from discontinuous controller, is handled by sliding with smooth control action.

KEYWORDS: DC motor, PID controller, Sliding mode controller (SMC)

ABSTRACT

In this thesis, sliding mode control (SMC) technique is used to control the speed of DC motor. The performance of the SMC is judged via MATLAB simulations using linear model of the DC motor and known disturbance. SMC is then compared with PID controller. The simulation result shows that the sliding mode controller (SMC) is superior controller than PID for the speed control of DC motor. Since the SMC is robust in presence of disturbances, the desired speed is perfectly tracked. The sliding mode control (SMC) can adapt itself to the parameter variations and external disturbances, problem of chattering parameter, resulting from discontinuous controller, is handled by sliding with smooth control action.

1. INTRODUCTION

DC motors have been widely used in many industrial applications such as electric vehicles, steel rolling mills, electric cranes, robotic manipulators, and home appliances due to precise, wide, simple, and continuous control characteristics. The purpose of a speed controller is to drive the motor at desired speed. DC motors are generally controlled by conventional Proportional plus Integral controllers, since they can be designed easily. However the performance of PID controller for speed control degrades under external disturbances and machine parameter variations. This makes the use of PID controller a poor choice for variable speed drive applications.

In the past three decades, nonlinear and adaptive control methods have been used extensively to control DC drives. In these methods, the state estimation and parameter identification are based on and limited to linear models. Performance comparison of sliding mode control and conventional pi controller for speed control of separately excited DC motors.

Here SMC, PID with SMC controller are designed for the DC motor system and their performance is compared.
II. MODELING OF DC MOTOR

A separately excited dc motor has the simplest decoupled electromagnetic structure [4]. A schematic diagram of the separately excited DC motor is shown in follows as description of the system along with the Mathematical model.

\[
L_a \frac{dI_a}{dt} + I_a R_a + E_b = E_a
\]  

(1)

Where \(E_a\) is the Applied Voltage, \(R_a\) is the armature resistance, \(L_a\) is the Equivalent armature inductance, \(I_a\) current flowing through armature circuit, \(E_b\) is the back emf and, the dynamics of the mechanical system is given by the torque balance equation

\[
J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} + T_i = T_m = K_i I_a
\]  

(2)

Terminal voltage \(V_a\) is taken to be the controlling variable. One can write state model with the \(\omega\) and \(I_a\) as state variables and \(V_a\) as manipulating variable, as given below

\[e_b(t) = k_b w(t)\]  

(3)

Where \(k_b\) is the back emf constant in \(V_s/\text{rad}\). The input terminal voltage \(V_a\) is taken to be the controlling variable. One can write state model with the \(\omega\) and \(I_a\) as state variables and \(V_a\) as manipulating variable, as given below

\[
x_1 = \omega
\]

\[
x_2 = x_1 = \dot{\omega} = w
\]

\[
x_3 = I_a
\]

\[
\dot{x} = \begin{bmatrix} \frac{W}{v_a} \\ \frac{-B}{J} x_2 - \frac{R_a}{J} x_3 \\ \frac{-K_t}{J} x_2 - \frac{R_a}{J} x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \\ \frac{1}{J} \end{bmatrix} V_a(t)
\]

(4)

PARAMETERS OF THE DC MOTOR

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SPECIFICATIONS</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_a)</td>
<td>Armature resistance</td>
<td>1.2 (\Omega)</td>
</tr>
<tr>
<td>(L_a)</td>
<td>Inductance of Armature winding</td>
<td>0.05H</td>
</tr>
<tr>
<td>(J)</td>
<td>Moment of inertia</td>
<td>0.135Kgm² (/s^2)</td>
</tr>
<tr>
<td>(B)</td>
<td>Frictional coefficient</td>
<td>0 Nms</td>
</tr>
<tr>
<td>(K_t)</td>
<td>Torque constant</td>
<td>0.06 Nm/A</td>
</tr>
<tr>
<td>(K_b)</td>
<td>Back emf constant</td>
<td>0.6 V</td>
</tr>
</tbody>
</table>

Using the parameters given in Table 1, transfer function of the DC motor with angular velocity as controlled variable and input terminal voltage as manipulating variable is determined as given below

\[
\frac{W(s)}{V_a(s)} = \frac{k_m}{s^2 + \left(\frac{R_a}{J} + \frac{L_a}{J} \right)s + \left(\frac{R_a}{J} + \frac{K_t}{J} \right)}
\]

(5)

(5) in time domain is as follows

\[
\frac{d^2 W}{dt^2} + \left(\frac{R}{J} + \frac{b}{J}\right) \frac{dW}{dt} + \left(\frac{R_a + K_m}{J}\right)W = \frac{k_m}{J} W
\]

(6)

However, if the state variables consider \(\begin{bmatrix} \omega \\ I_a \end{bmatrix} = \begin{bmatrix} w \\ I_a \end{bmatrix}\) and \(\begin{bmatrix} \omega \\ I_a \end{bmatrix} = \begin{bmatrix} w \\ I_a \end{bmatrix}\). The system described by equation (4) by equation (8) will be expressed, Where the only variable is the angular velocity and derivative

\[
\dot{x} = \begin{bmatrix} 0 \\ A_1 \\ A_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} w \\ I_a \end{bmatrix}[u(t)]
\]

(8)

\[
A_1 = -\left(\frac{R_a + K_m}{J}\right)
\]

\[
A_2 = \left(\frac{8}{J} + \frac{b}{J}\right)
\]

(9)

IV. SLIDING MODE CONTROLLER DESIGN

A linear system can be described in the state space as follows

\[
\dot{x} = Ax + Bu
\]

(12)

Where \(X \in R^n\), \(U \in R\), \(A \in R^{n \times n}\), \(B \in R^n\) is a full rank matrix Where A and B are controllable matrices and the functions of state variables are known as switching function
\[ \sigma = S x \]  
(13)

The main idea in sliding mode control is
• Designing the switching function so that manifold (sliding mode) provide the desired dynamic (\( \sigma = 0 \))
• Finding a controller ensuring sliding mode of the system occurs in finite time
First of all, the system should be converted to its regular form
\[ X = T x \]  
(14)

Where \( T \) is the matrix that brings the system to its regular form

\[ \dot{x}_1 = A_{11} x_1 + A_{12} x_2 \]  
\[ \dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_2 u \]  
(15)

The switching function in regular form is:
\[ \sigma = s_1 \dot{x}_1 + s_2 \dot{x}_2 \]  
(16)

On the sliding mode manifold (\( \sigma = 0 \))

\[ \dot{x}_2 = s_2^{-1} s_1 \dot{x}_1 \]  
(17)

From (17) & (15)

\[ \dot{x}_1 = A_{11} x_1 + A_{12} s_2^{-1} s_1 \dot{x}_1 \]  
(18)

One of matrixes in product: \( s_2^{-1} s_1 \) should be chosen arbitrary. Usually (19) is used to ensure that \( S2 \) is invertible
\[ \dot{x}_2 = B_2^{-1} \]  
(19)

can be calculated by assigning the Eigen value of (18) by pole placement method. Hence, switching function will be obtained as follows:
\[ S = [s_1 \ s_2]^T \]  
(20)

The control rule is:
\[ u = u_c + u_d \]  
(21)

Where \( u_c \) and \( u_d \) are continuous and discrete parts, respectively and can be calculated as follows:
\[ u_c = - \tilde{A}_{21} x_1 - \tilde{A}_{22} \sigma \]  
(22)

\[ u_d = - k_s \text{sign}(\sigma) - k_p(\sigma) \]  
(23)

Where sign is sign function. \( \sigma \) and \( w \) are constants calculated regarding to Lyapunov stability function

We are going to set the angular velocity over a certain value \( r \), so switching function is
\[ \sigma = s_1 (x_1 - r) + s_2 x_2 \]  
(24)

If the controller switching function is designed to be placed on the surface \( \sigma = 0 \), then solving equations (24) assume \( \sigma = 0 \) and \( w \) and \( w_d \) are obtained by

\[ w = r \left(1 - e^{\frac{(s_1)}{s_2}}t\right) \]  
(25)

\[ \dot{w} = r \left(\frac{v_1}{s_2} - \frac{v_2}{s_2}t\right) \]  
(26)

As equation (8) it is regular form, so the transformation matrix is equal to the unit matrix

\[ s_2 = \frac{J_L}{K_m} \]  
(27)

Also according to (12-19) \( 1 \ s \) calculated and \( w \) Pole placement method using (12-21). Suppose we have to placed system poles \( \lambda \) in so we have

\[ \frac{s_1}{s_2} = \lambda \]  
(28)

\[ \sigma = \frac{J_L}{K_m} - \lambda (w - r) + \dot{w} \]  
(29)

A. CONTROLLER DESIGN:

If the equation (8) can be rewritten based on the state variables and \( x_i = (\dot{x}_1 - r) \) the following is reached

\[ [\dot{x}_1 \ \dot{x}_2 \ \sigma] = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 1 \\ 0 & 0 & 0 \end{bmatrix} [x_1 \ \sigma] + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_n \]  
(20)

\[ \tilde{A}_{21} = A_{21} - \frac{s_1}{s_2} \]  
(21)

\[ \tilde{A}_{22} = A_{22} + \frac{s_1}{s_2} \]  
(22)

\[ u_n = s_2^{-1} u + A_{11} r \]  
(23)

Thus the relations (21), (22) and (23) controller for the system (20) is designed as follows

\[ u_n = - \tilde{A}_{21} x_1 - \tilde{A}_{22} \sigma - k_s \text{sign}(\sigma) - k_p(\sigma) \]  
(24)

The below equation Sets armature voltage feedback based on the derivative of the angular velocity for motor.

\[ u = S_2 [A_r + S_2 (A_1 + A_2 \lambda - \lambda^2) (w - r) + (A_2 - \lambda) \sigma + k_s \text{sign}(\sigma) + k_p(\sigma) \]  
(25)

So the sliding mode controller is

\[ u = \frac{J_L}{K_m} \left(\frac{Rb + K_n K_m}{J_L} \right) w + \frac{J_L}{K_m} \left(\frac{Rb + K_n K_m}{J_L} \right) \lambda \left(\frac{R}{L} + \frac{b}{J_L} \right) + \lambda^2 \left(\frac{R}{L} + \frac{b}{J_L} \right) \lambda \cdot k_s \text{sign}(\sigma) - k_p(\sigma) \]  
(26)
Switching function of sliding mode controller for DC motor control method according to the relations (34) and (33) are designed. If the motor parameters like table (1), then the controller will numerically designed as follows

\[
\sigma = -1.12(w - r) + 0.0112 \dot{w} 
\]  

(35)

After solving the controller \( u \) is given by

\[
U = 0.0112(53.24)w + 0.0112(53.25)(w - r) - 124(\sigma) - sgn(s)
\]  

(36)

Where \( \lambda, k_s \) and \( k_p \) parameters are \(-100, 1\) and \(0\) respectively

V. RESULTS AND OUTPUTS

The DC motor, a PID controller is attached and the corresponding simulink model and its output for the same reference input of 1000rpm is given below

The DC motor, a SMC controller is attached and the corresponding simulink model and its output for the same reference input of 1000rpm is given below

Comparing the PID, SMC, SMC with PID is attached to DC motor with a reference speed of 1000 rpm. The outputs are compared based on the settling time.
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Figure 6: Simulink model of DC motor of combined with PID, SMC, SMC with PID

Figure 7: Speed response of DC motor is done by using PID controller and SMC, the response due to SMC is better compared to PID controller.
SMC does not vary with parameter variations, by varying the parameters of R, L, J of DC motor, by increasing the percentage of R, L, J parameters of DC motor with speed of 1000 rpm.

Figure 8: Comparison of internal parameters of DC motor

Figure 9: Speed response of DC motor is observed by varying the internal parameters R, L, J of the DC motor.

Figure 10: Simulink model of DC motor is observed when external disturbance is added.

Figure 11: Speed response of DC motor is observed when external disturbance is added.

**COMPARISON TABLE**

<table>
<thead>
<tr>
<th>Controller</th>
<th>$T_s$ (sec)</th>
<th>OVERSHOOT</th>
<th>DISTURBANCE REJECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>0.3</td>
<td>PRESENT</td>
<td>POOR</td>
</tr>
<tr>
<td>SMC</td>
<td>0.2</td>
<td>NIL</td>
<td>GOOD</td>
</tr>
<tr>
<td>SMC WITH PID</td>
<td>0.07</td>
<td>NIL</td>
<td>GOOD</td>
</tr>
</tbody>
</table>

Where $T_s$ = settling time

**VI. CONCLUSION**

In this paper sliding mode control (SMC) Proposed to speed control of DC motor. At first for controlling speed of DC motor a simplified closed loop is utilized. Then DC motor is modeled after that speed controller is designed. As sliding mode control is based on the system Dynamic characteristics also it took a lack of influence of external disturbances from user as result it worked more useful and results confirms that used sliding mode control for speed control is more efficient in comparison with PID controller.

**ACKNOWLEDGMENT**

The authors would like to express their gratitude to Dr. S.V.H Rajendra, Secretary, AlwarDas Group of Educational Institutions, SriVBhaskar, Dean for...
their encouragement and support throughout the course of work. The authors are grateful to Dr. N.C. Anil, Principal, Sanketika Institute of Technology and Management, EEE Department and staff for providing the facilities for publication of the paper.

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